Section 5.1
Functions, Domains and Ranges

A **relation** is a set of ordered pairs.

A **function** is a set of ordered pairs {(x, y)} in which no x-value is repeated.

Example 1: Determine whether each relation is a function.

a. {(1, 0), (2, 3), (3, 4), (4, 0)}

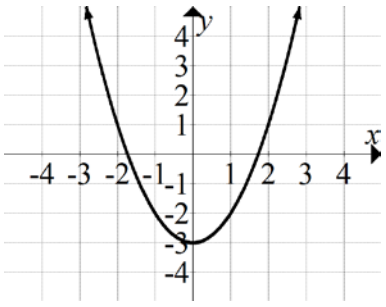
b. {(1, -1), (1, 3), (-1,1)}

c. {(0, 10), (1, 10), (2,10)}

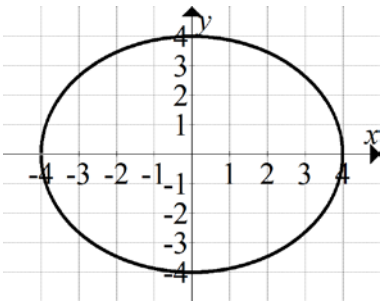
We can also look at a graph and easily determine if it is the graph of a function. We can use the **vertical line test** to accomplish this. The vertical line test states that the relation graphed is a function so long as no vertical line can intersect the graph in more than one point.

Example 2: Determine whether each relation is a function.

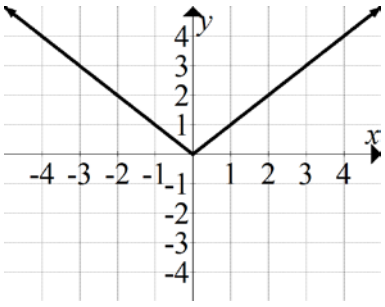
a.



b.



c.



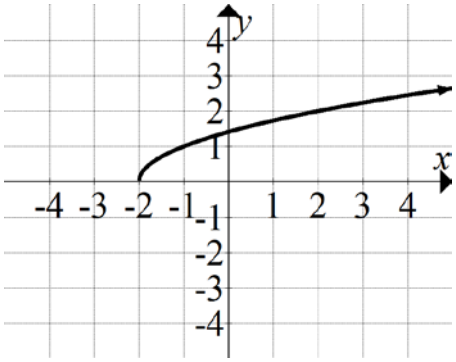
Domain and Range

For a function, each ordered pair has a unique x-value. The set of these x-values is called the **domain** of the function. The set of all the y-values is called the **range** of the funciton.

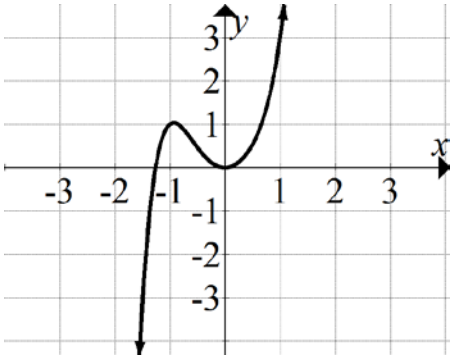
Example 3: Recall the set: {(1, 0), (2, 3), (3, 4), (4, 0)} which was a function. Give the domain and range of the function.

Example 4: Give the domain and range of each function.

a.



b.



Most of the time, the functions are given by formulas, rather than specifying a set of ordered pairs. In this case, we must be careful to exclude from the domain all x-values for which the function is undefined.

Two important cases:

1. x-values which lead to **division by zero** should be excluded from the domain.

2. x-values which lead to **an even root of a negative number** should be excluded from the domain.

Example 5: Find the domain of each function.

a. $f(x) = \dfrac{x-9}{x}$

b. $f(x) = \dfrac{5x}{3x-8}$

c. $f(x) = \sqrt{-2x}$

d. $f(x) = \sqrt{7x+1}$

e. $f(x) = \dfrac{-3x}{x^2 + 9}$

f. $f(x) = \dfrac{1}{\sqrt{x+5}}$

g. $f(x) = \dfrac{\sqrt{-x+1}}{x+2}$

Function Values

Once the domain of a function is known, we can calculate function values. A **function value** is a corresponding y-value, given an x-value.

Example 6: Let $f(x) = \dfrac{20}{\sqrt{x-4}}$ and $g(x) = 4x^{-3}$ find $f(5) + g\left(\dfrac{1}{3}\right)$.