Section 3.3
Geometric Distributions

The **geometric distribution** is the distribution produced by the random variable X defined to count the number of trials needed to obtain the first success.

For example: Flipping a coin until you get a head. Rolling a die until you get a 5

A random variable X is geometric if the following conditions are met:

1. Each observation falls into one of just two categories, “success” or “failure.”

2. The probability of success is the same for each observation.

3. The observations are all independent.

4. The variable of interest is the number of trials required to obtain the first success.

Notice that this is **different from the binomial distribution in that the number of trials is unknown**. With geometric distributions we are trying to determine how many trials are needed in order to obtain a success.

Geometric formula to find the probability that the first success occurs on the n^{th} trial is
$P(X = n) = (1-p)^{n-1} \cdot p$ where p is the probability of success.

R command: $P(X = n)$ =dgeom(n – 1, p)

Example 1: A quarterback completes 44% of his passes. We want to observe this quarterback during one game to see how many pass attempts he makes before completing one pass. What is the probability that the quarterback throws 3 incomplete passes before he has a completion?

Command: Answer:

Geometric formula to find the probability that it takes *more* than n trials to see the first success is $P(X > n) = (1-p)^n$.

R commands:
$P(X \le n)$ = pgeom(n – 1, p)
$P(X > n)$ = 1 – pgeom(n – 1, p)

Example 2: There is a probability of 0.08 that a vaccine will cause a certain side effect. Suppose that a number of patients are inoculated with the vaccine. Find the probability that
a. more than 5 patients must be vaccinated in order to observe the first side effect.

Command: Answer:

b. 7 or fewer patients must be vaccinated in order to observe the first side effect.

Command: Answer:

Geometric Distribution Mean, Variance and Standard Deviation Formulas

$E(X) = \mu = \frac{1}{p}$.

Variance: $\sigma^2 = \frac{1-p}{p^2}$

Standard Deviation: $\sigma = \sqrt{\frac{1-p}{p^2}}$

Example 3: Newsweek in 1989 reported that 60% of young children have blood lead levels that could impair their neurological development. Assuming a random sample from the population of all school children at risk, how many students would you expect to have tested in order to find the first one to have these blood levels?

Recall: For a geometric distribution we are trying to determine how many trials are needed in order to obtain a success, and that the number of trials is unknown. This is how it differs from a binomial distribution.

Example 4: Indicate whether the following distribution is geometric or binomial.
a. Roll a pair of dice until you get doubles.
Geometric or Binomial

b. Roll a die 10 times and observe the number of times a one comes up.
Geometric or Binomial

Try this one: You roll a die 8 times. Find the probability that a three comes up twice.

Command: Answer:

Example 6: Find the probability a die must be rolled more than 4 times before a three comes up.

Command: Answer: