Discrete Mathematics
Sets

Definition: A *set* is an unordered collection of distinct objects.
Examples:

a. the students in this class
b. the chairs in this room
c. counting numbers

Definition: The objects in a set are called the *elements*, or *members* of the set. A set is said to *contain* its elements.
Notation:

- The notation $a \in A$ denotes that a is an element of the set A.
- If a is not a member of A, write $a \notin A$.

Ways to describe a set

1. Roster method
    - List all elements of a set
    - Order NOT important
    - Each distinct object is either a member or not; listing more than once does not change the set.
    - Ellipses (…) may be used to describe a set without listing all of the members when the pattern is clear.

    Example: $S = \{a, b, c, d\}$

Usually elements in a set have a common property but this is NOT necessary.

Example: $C = \{\text{I, love, math, 10, Houston}\}$

2. Set builder notation
    - Specify a property that all elements in a certain set must have.

Example: $B = \{x \mid x \text{ is an even positive integer less than } 12\}$

Some important sets

$\mathbb{N} = \textit{natural numbers} = \{0,1,2,3 \ldots.\}$

$\mathbb{Z} = \textit{integers} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

$\mathbb{Z}^{+} = \textit{positive integers} = \{1,2,3, \ldots..\}$

$\mathbb{R}$ = set of *real numbers*

$\mathbb{R}^{+}$ = set of *positive real numbers*

$\mathbb{C}$ = set of *complex numbers*.

$\mathbb{Q}$ = set of *rational numbers*

Example: $B = \{x \mid x \text{ is an even positive integer less than } 12\}$

Example: $\{\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}\}$

Definition: Two sets are equal if and only if they have the same elements.

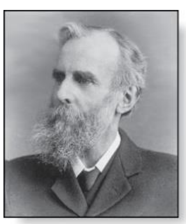
Notation: $A = B \ iff \ \forall x \ (x \in A \leftrightarrow x \in B)$

Example: $A = \{red, blue, black\}, B = \{\ black, red, blue\}$

Example: $A = \{red, red, red, red\}, B = \{red\}$

Venn Diagrams

Sets can be represented graphically using Venn diagrams. Venn diagrams are often used to show relations between sets.



John Venn (1834-1923)
Cambridge, UK

The *universal set U* is the set containing everything currently under consideration.

- Sometimes implicit
- Sometimes explicitly stated.
- Contents depend on the context.

The *empty (null) set* is the set with no elements. Symbolized ∅, but {} also used.

Example: Draw a Venn diagram that represents $V = \{a, e, i, o, u\}$.

Example: $A = \{x \in \mathbb{N} \,|x < 0\}$

Definition: A set with one element is called a singleton set.

Example: $A = \{x \in \mathbb{N}|x \leq 0\}$

Definition: The set A is a *subset* of B if and only if every element of A is also an element of B.

A is a *strict (proper) subset* of B if A is a subset of B and A is not equal to B.

Notation: $A \subseteq B$ and $A \subset B$ respectively.

Example: $\{1, 2\} \subset \{1, 2, 3, 4\} \subseteq \{1, 2, 3, 4\}$

- To show that $A \subseteq B$, show that if x belongs to A, then x also belongs to B.
- To show that $A \nsubseteq B$, find a single $x \in A$ such that $x \notin B$.
- For every set S, $\emptyset \subset S$ and $S \subseteq S$.

Prove: For every nonempty set S, $\emptyset \subset S$.

Proof:

Definition: Given a set A, cardinality (size) of A is the number of elements in A.

Notation: $|A|$

Definition: The power set is the set of all subsets of a given set, and is denoted by $\mathcal{P}(A)$.

If a set has n elements, its power set contains 2^n elements.

Try this:

$A = \{1, \{2\}, \{3,4\}\}$, $B = \{1, 2, 3, 4\}$

1. True or False?
    a. $1 \in A$
    b. $2 \in A$
    c. $\{3\} \subset A$
    d. $2 \in B$
    e. $\{3\} \subset B$
    f. $\emptyset \in A$
    g. $\emptyset \subset B$
    h. $A \subseteq B$

2. Find
    a. $|A|$
    b. $|B|$
    c. $\mathcal{P}(A)$
    d. $|\emptyset|$
    e. $|\{\emptyset\}|$
    f. $\mathcal{P}(\emptyset)$
    g. $\mathcal{P}(\{\emptyset\})$