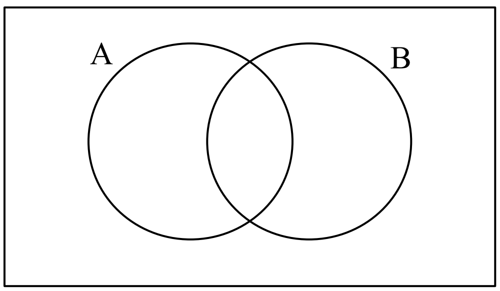
Discrete Mathematics
Set Operations

Definition: Let A and B be sets. The *union* of the sets A and B, denoted $A \cup B$, is the set that contains those elements that are either in A or in B, or in both.

Try this one: Write the union of A and B in the set builder notation.



Venn diagram for $A \cup B$

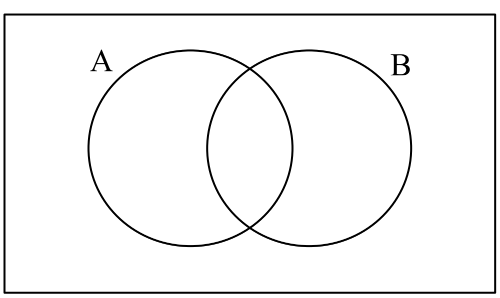
Definition: Let A and B be sets. The *intersection* of the sets A and B, denoted, $A \cap B$, is the set containing those elements in both A and B.
Two sets are called *disjoint* if their intersection is empty.

Try this one: Write the intersection of A and B in the set builder notation.

Example 1:

a. Find $\{5, 7, 19\} \cup \{1, 2, 3\}$.

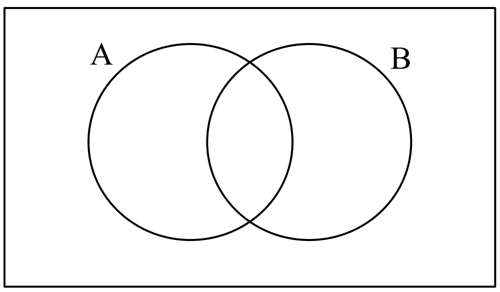
b. Find $\{5, 7, 19\} \cap \{1, 2, 3\}$.



Venn diagram for $A \cap B$

Definition: Let A and B be sets. The *difference* of A and B, denoted by $A - B$, is the set containing the elements of A that are not in B. The difference of A and B is also called the complement of B with respect to A.

Try this one: Write the difference of A and B in the set builder notation.

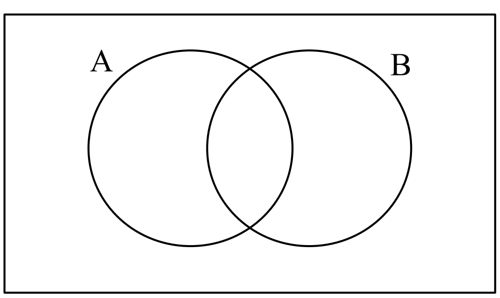


Venn diagram for $A - B$

Example 2: Find $\{5, 7, 19\} - \{1, 2, 3\}$

Definition: Let U be the universal set. The complement of the set A, denoted by $\bar{A}$, is the complement of A with respect to U. Therefore, the complement of the set A is $U - A$.

Try this one: Write $\bar{A}$, in the set builder notation.



Venn diagram for $\bar{A}$

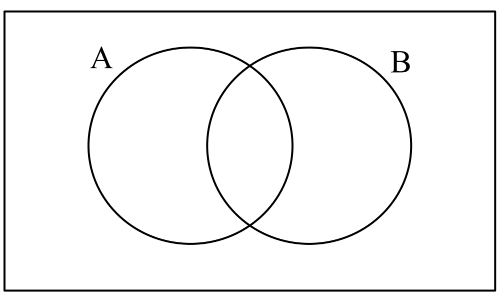
Example 3: Let A be the set of positive natural numbers and the universal set is the set of natural numbers. Write $\bar{A}$.

The cardinality of the union of two sets

$|A \cup B| = |A| + |B| - |A \cap B|$ Inclusion-exclusion principle

Example 4: Find $|A \cup B|$ if $A = \{1,2,3,4,5\}$ and $B = \{2, 3, 4, 5,6,7,8\}$.

Definition: Let A and B be sets. The *symmetric difference* of A and B, denoted by $A \oplus B$ is the set $(A - B) \cup (B - A)$.



Venn diagram for symmetric difference

Try this one: $U = \{x \in \mathbb{N} \mid x \leq 10\}$, $A = \{1,2, 3\}$, and $B = \{3, 4, 5\}$. Find $A \oplus B$.

Set Identities

- Identity laws: $A \cup \emptyset = A$; $A \cap U = A$
- Domination laws: $A \cup U = U$; $A \cap \emptyset = \emptyset$
- Idempotent laws: $A \cup A = A$; $A \cap A = A$
- Complementation law: $\overline{(\bar{A})} = A$
- Commutative laws: $A \cup B = B \cup A$; $A \cap B = B \cap A$
- Associative laws: $A \cup (B \cup C) = (A \cup B) \cup C$; $A \cap (B \cap C) = (A \cap B) \cap C$
- Distributive laws: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$;

  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- De Morgan's laws: $\overline{A \cup B} = \bar{A} \cap \bar{B}$; $\overline{A \cap B} = \bar{A} \cup \bar{B}$
- Absorption laws: $A \cup (A \cap B) = A$; $A \cap (A \cup B) = A$
- Complement laws: $A \cup \bar{A} = U$; $A \cap \bar{A} = \emptyset$

Proving set identities

Different ways to prove set identities

- Prove that each set (side of the identity) is a subset of the other.

- Membership Tables: Verify that elements in the same combination of sets always either belong or do not belong to the same side of the identity. Use 1 to indicate it is in the set and a 0 to indicate that it is not.

Example 5: Construct a membership table to show $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Example 6: Prove $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ by showing that RHS set and LHS set are subsets of each other.

Definition: Let A and B be sets. The Cartesian product of A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$.

Try this: Write $A \times B$ in the set builder notation.

Example 7: Let $A = \{1,2,3\}$ and $B = \{a, b\}$. Find $A \times B$ and $B \times A$.

Example 8: Find A^3 if $A = \{a\}$.

Example 9: How many different elements does $A \times B$ have if A has m elements and B has n elements?

Example 10: Suppose $A \times B = \emptyset$, where A and B are sets. What can you conclude?