MATH 3336 – Discrete Mathematics
Combinations and Permutations (6.3)

Permutations

Definition: A *permutation* of a set of distinct objects is an ordered arrangement of these objects. An ordered arrangement of r elements of a set is called an *r-permutations.*

Notation: The number of *r-permutations* of a set with n elements is denoted by $P(n, r)$.

Example: Let $S = \{1,2,3\}$.

- The ordered arrangement 3, 1, 2 is a permutation of S.
- The ordered arrangement 3, 2 is a 2-permutation of S.

Write all 2-permutations of $S = \{1,2,3\}$.

Theorem: If n is a positive integer and r is an integer with $1 \leq r \leq n$, then there are

$$P(n, r) = n(n - 1)(n - 2) \cdots (n - r + 1)$$

r-permutations of a set with n distinct elements.

Proof:

Corollary: If n and r are integers with $1 \leq r \leq n$, then

$$P(n, r) = \frac{n!}{(n-r)!}$$

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Solving Counting Problems by Counting Permutations

Example: How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest?

Example: Suppose that a saleswoman has to visit eight different cities. She must begin her trip in a specified city, but she can visit the other seven cities in any order she wishes. How many possible orders can the saleswoman use when visiting these cities?

Example: How many permutations of the letters *ABCDEFGH* contain the string *ABC*?

Combinations

Definition: An *r-combination* of elements of a set is an unordered selection of *r* elements from the set. Thus, an *r*-combination is simply a subset of the set with *r* elements.

Notation: The number of *r*-combinations of a set with n distinct elements is denoted by $C(n, r)$. This notation is also used and is called a *binomial coefficient*.

Example: Let S be the set $\{1, 2, 3\}$. Then $\{1, 2\}$ is a 2-combination from S. It is the same as $\{2, 1\}$ since the order listed does not matter.

Find $C(3,2)$.

Theorem: The number of *r*-combinations of a set with n elements, where $0 \le r \le n$, equals

$$C(n, r) = \frac{n!}{r!\,(n-r)!}$$

Proof:

Solving Counting Problems by Counting Combinations

Example: How many poker hands of five cards can be dealt from a standard deck of 52 cards? Also, how many ways are there to select 47 cards from a deck of 52 cards?

Corollary: Let n and r be nonnegative integers with $r \leq n$. Then $C(n, r) = C(n, n - r)$.

Proof:

Example: Find the following values:

$C(n, 1) =$

$C(n, 0) =$

$C(n, n) =$

$C(n, n - 1) =$

Example: How many ways are there to select six players from a 10-member tennis team to make a trip to a match at another school?

Example: A judge has a jury pool of 40 people that contains 22 women and 18 men. She needs a jury of 12 people.

a. How many juries can be made?

b. How many juries contain 6 women and 6 men?

Example: A customer at a fruit stand picks a sample of 7 oranges at random from a crate containing 35 oranges of which 5 are rotten.

a. How many selections can be made?

b. How many selections contain 4 rotten?

c. How many selections contain at least 4 rotten?

d. How many selections contain at most 4 rotten?

e. How many selections contain at least 1 rotten?