NUMERICAL ANALYSIS

Math 4364 (Fall 2011)

September 6, 2011

This exam has 3 questions, for a total of 100 points.

Please answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Name and ID:

First Test

Solution

Keys

30 points | 1. Let

$$A = \begin{pmatrix} 0 & 2 & 3 \\ 1 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

- a) Determine the LU factorization of the form PA = LU for the matrix A.
- b) Use the factorization to solve the system

$$2x_{2}+3x_{3}=5$$

$$x_{1}+x_{2}-x_{3}=1$$

$$-x_{2}+x_{3}=0$$
a) $(E_{1}) \leftarrow (E_{2})$; $M_{2,1}=0$, $M_{3,1}=0$

$$(E_{3}+\frac{1}{2}E_{2}) \rightarrow (E_{3})$$

$$(E_{3}+\frac{1}{2}E_{2}) \rightarrow (E_{3})$$

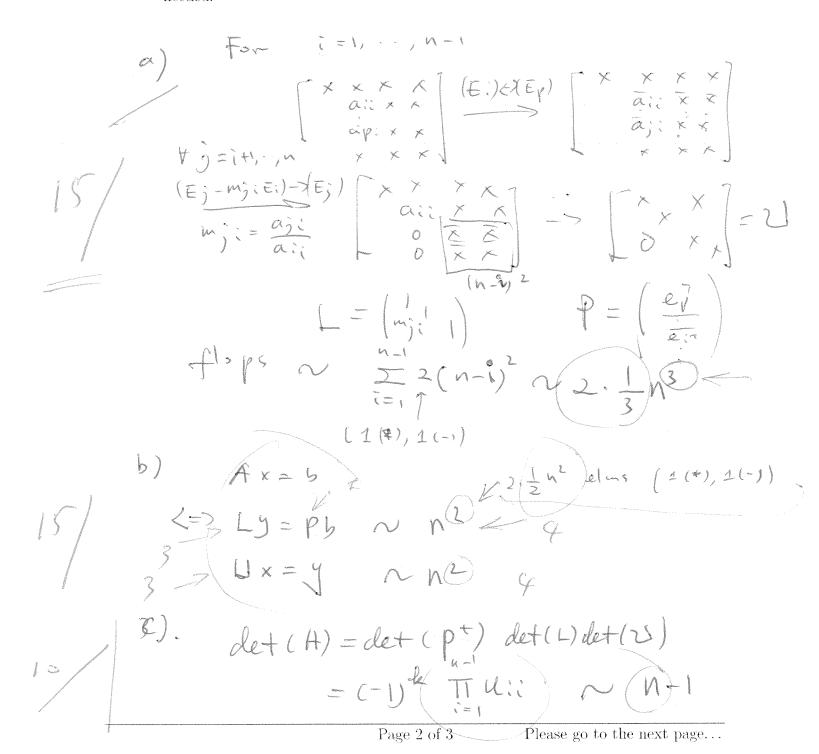
$$A = b$$

Page 1 of 3

Please go to the next page...

40 points

- 2. Let A be an $n \times n$ nonsingular matrix and b be an n column vector.
 - a) Determine the number of operations needed to perform the LU factorization of the form PA = LU for the matrix A.
 - b) Use the factorization to solve the system Ax = b and determine the number of operations needed.
 - c) Use the factorization to compute $\det A$ and determine the number of operations needed.



30 points

3. Let

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

40

a) Determine the LU factorization of the form A = LU for the matrix A and show that A is positive definite.

 \nearrow b) Use the LU factorization to determine the LDL^t factorization of A.

 \mathbb{R}^{-1} c) Use the LDL^t factorization to determine the Cholesky factorization of A.

a) col1.
$$M_{21} = -\frac{1}{2}$$
, $M_{51} = 0$

$$(E_{2} + \frac{1}{2}E_{1}) \rightarrow (E_{L})$$

$$(= 1 - \frac{1}{2}) \rightarrow (E_{L})$$

$$L = \begin{bmatrix} 2 & -1 & 0 \\ 0 & \frac{3}{2} & -1 \\$$

2)
$$A = L D L^{\dagger}$$
, $D = diag(di) = \begin{bmatrix} 2 & 0 & 0 \\ 0 & \frac{3}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$

$$L = L \operatorname{diag}(Nd:) = \begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{2}{3} & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2} & \sqrt{6} & 0 \\ 0 & \sqrt{2} & 2 \end{bmatrix}$$

When you finish this exam, you should go back and reexamine your work for any errors that you may have made.

End of exam.