NUMERICAL ANALYSIS

Add. Sample Pbs Math 4365 (Spring 2012) April 24, 2012

25 points 1. What straight line best fits the data

x	1	2	3	4
y	0	1	1	2

in the least-squares sense?

25 points 2. Let g_0, g_1, ..., g_n be a set of functions such that

$$\sum_{k=0}^{m} g_i(x_k)g_j(x_k) = 0, \quad \forall i \neq j$$

What linear combination of these function best fits the following data

x	x_0	x_1	$\cdots$	x_n
y	y_0	y_1	$\cdots$	y_n

in the least-squares sense?

25 points 3. Let $\{\phi_0, \phi_1, \ldots, \phi_{2n-1}\}$ be a set of functions such that

$$\begin{aligned}
\phi_0(x) &= \frac{1}{2}, \\
\phi_k(x) &= \cos kx, \quad \forall k = 1, 2, \cdots, n, \\
\phi_{n+k}(x) &= \sin kx, \quad \forall k = 1, 2, \cdots, n-1,
\end{aligned}$$

What linear combination of these function best fits the following data

x	x_0	x_1	$\cdots$	x_{2m-1}
y	y_0	y_1	$\cdots$	y_{2m-1}

where

$$x_j = -\pi + \left(\frac{j}{m}\right)\pi, \quad \forall j = 0, 1, \cdots, 2m-1$$

in the least-squares sense?

25 points 4. Apply the Finite-Difference method to solve

$$\begin{aligned}
&\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 1, \quad 0 < x < 1, \quad 0 < y < 1, \\
&u(x, 0) = 0, \quad u(x, 1) = 0, \quad 0 \leq x \leq 1, \\
&u(0, y) = 0, \quad u(1, y) = 0, \quad 0 \leq y \leq 1,
\end{aligned}$$

and write down the detailed algorithm.