NUMERICAL ANALYSIS

Sample Test 2 **Math 4365 (Spring 2012)** **February 16, 2012**

$\boxed{\text{20 points}}$ 1. Taylor's theorem can be used to show that centered-difference formula to approximate $f'(x_0)$ can be expressed with an error formula

$$f'(x_0) = \frac{1}{2h}\left[f(x_0+h) - f(x_0-h)\right] - \frac{h^2}{6}f'''(x_0) - \frac{h^4}{120}f^{(5)}(x_0) - \cdots$$

Find approximations of order $O(h^2)$, $O(h^4)$, and $O(h^6)$ for $f'(2)$ when $h = 0.2$ and $f(x)$ is represented by the following table

x	1.8	1.9	1.95	2.05	2.1	2.2
$f(x)$	10.889	12.703	13.706	15.924	17.149	19.855

2. (a) Use Simpson's rule to approximate

$$\int_1^{1.5} x^2 \ln x dx$$

(b) Find a bound for the error in the Simpson's rule approximation in part (a).

$\boxed{\text{20 points}}$ 3. A car laps a race track in 60 seconds. The speed of the car at each 6-second interval is determined by using a radar gun and is given from the beginning of the lap, in feet/second, by the entries in the following table. Use the Composite Simpson's rule to determine the length of the track.

Time	0	6	12	18	24	30	36	42	48	54	60
Speed	124	134	148	156	147	133	121	109	99	85	78

$\boxed{\text{20 points}}$ 4. Determine constants a, b, c and d that will produce a quadrature formula

$$\int_{-1}^{1} f(x)dx = af(-1) + bf(1) + cf'(-1) + df'(1)$$

that has degree of precision 3.

$\boxed{\text{20 points}}$ 5. Use the Composite Simpson's rule for $n = 4$ to approximate the value of the improper integral

$$\int_0^1 x^{-1/4} \sin x dx$$