NUMERICAL ANALYSIS

Sample Test 3 **Math 4365 (Spring 2012)** **March 6, 2012**

40 points

1. Consider the initial value problem

$$y' = -2y + te^{3t}, \quad 0 \le t \le 1, \quad y(0) = 0. \tag{1}$$

   (a) Use Euler's method with $h = 0.5$ to approximate the solution to equation (1).

   (b) The exact soltuion to the initial value problem (1) is

$$y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}$$

   Determine an error bound for the approximation obtained in (a).

   (c) Use Taylor's method of order two with $h = 0.5$ to approximate the solution to equation (1)

   (d) Use the modified Euler method with $h = 0.5$ to approximate the solution to equation (1).

   (e) Use the Runge-Kutta method of order four with $h = 0.5$ to approximate the solution to equation (1).

40 points

2. Consider the following Runge-Kutta method

$$\begin{aligned}
&w_0 = y_0, \\
&\text{for } i = 0, 1, \cdots, N-1, \\
&\quad k_1 = hf(t_i, w_i), \\
&\quad k_2 = hf(t_i + \alpha h, w_i + \beta k_1) \\
&\quad w_{i+1} = w_i + a_1 k_1 + a_2 k_2
\end{aligned}$$

   (a) Show that the above Runge-Kutta method cannot have local truncation error $O(h^3)$ for any choice of constants a_1, a_2, α and β.

   (b) Show that the above Runge-Kutta method is of order 2 if, for any α,

$$\beta = \alpha, \quad a_1 = 1 - \frac{1}{2\alpha}, \quad a_2 = \frac{1}{2\alpha}.$$

   (c) Show that by chosing $\alpha = 1$ in (b), we obtain the modified Euler method.

   (d) Show that by chosing $\alpha = \frac{1}{2}$ in (b), we obtain the midpoint method.

20 points

3. Derive the Adams-Bashforth two step method by using the Lagrange form of the interpolating polynomial.