$$\lim_{x\to0^+} \frac{(1+x)^{\frac{1}{x}} - e}{8x} \overset{\text{L.H.}}{=} \lim_{x\to0^+} \frac{\frac{1}{x}\boxed{(1+x)^{\frac{1}{x}-1}} - \ln(x+1)\,(1+x)^{1/x}/x^2}{8}$$

log. diff.

"$\frac{0}{0}$" ind. form

$$= \lim_{x\to0^+} \frac{(1+x)^{\frac{1}{x}-1}}{8} \cdot \left[\frac{1}{x} - \frac{\ln(x+1)\,(1+x)}{x^2} \right]$$

$$= \frac{e}{8} \lim_{x\to0^+} \underbrace{\left[\frac{x - \ln(x+1)(1+x)}{x^2} \right]}_{\text{"}\frac{0}{0}\text{" ind. form}}$$

L.H.

$$= \frac{e}{8} \lim_{x\to0^+} \frac{1 - (1 + \ln(x+1))}{2x} \overset{\text{simplified}}{=} \frac{e}{8} \lim_{x\to0^+} \frac{-\ln(x+1)}{2x}$$

"$\frac{0}{0}$" ind. form

L.H.

$$= \frac{e}{8} \lim_{x\to0^+} \frac{-\frac{1}{x+1}}{2} \to -\frac{1}{2}$$

$$= -\frac{e}{16}$$