June 23, 2020

Name:

Final Math 5336

1. Determine whether the relation R on the set of all real numbers is reflexive, symmetric, and/or transitive, where $(x,y) \in R$ if and only if
   (a) $x + y = 0$;
   (b) $x = \pm y$;
   (c) $x - y$ is a rational number;
   (d) $x = 1$ or $y = 1$
2. What is the largest equivalence relation $R \subseteq A \times A$ on the set A and what is the smallest equivalence relation on the set A?
3. Define that π is a partition of the non-empty set A. How are equivalence classes of an equivalence relation defined? What is the partition of equality?
4. (a) Is the union of two equivalence relations R and S on the set A and equivalence relation?
   (b) Is the intersection of two equivalence relations R and S an equivalence relations?
   You must prove your answers.
5. Let $A = \{a,b,c,d,e,f,g\}$. What are the classes of the smallest equivalence relation that contains the following pairs $\{(a,c),(e,c),(d,f),(g,d),(b,e)\}$?
6. (a) Let $f : A \to B$ be any function from the set A to the set B. Define a relation R_f on A by aRb if and only if $f(a) = f(b)$. Explain why R_f is an equivalence relation.
   (b) Assume that $f : A \to B$ is a surjective function. How can you define a bijection from the equivalence classes for R_f onto B?
7. (a) Let $A = \{a,b,c,d,e,f,g\}$ and $B = \{1,2,\}$ and let f be the function for which one has that $f(a) = 1, f(b) = 1, f(c) = 1, f(d) = 2, f(e) = 1, f(f) = 2, f(g) = 2$. What is the partition of the equivalence relation R_f for f?
   (b) Let $f : A \to B$ be a surjection from A to B. Assume that A has n−many elements and B has m−many elements. What can you say about the number k of equivalence classes for the equivalence relation R_f?
      i. $k = n$;
      ii. $k = m$; iii. none of the above.
8. Define that P is a partial order of the set A.
9. Give an example of a partial order which is not a total order.
10. The theorem of Cantor-Bernstein says that if there is an injection f from the A into the set B and an injection g from the set B into the set A then there is a bijection h from the set A to the set B. Using this theorem prove that there is a bijection between the open interval $(0,1)$ and the closed interval $[0,1]$.