§ 1.2 The Mean, Variance, and Standard Deviation

mean: $\mu = \sum_{x\in S} xf(x) = u_1 f(u_1) + ... + u_k f(u_k)$

variance: $\sigma^2 = \sum_{x\in S} (x-\mu)^2 f(x) = (u_1-\mu)^2 f(u_1) + ... + (u_k-\mu)^2 f(u_k) = \sum_{x\in S} x^2 f(x) - \mu^2$

standard deviation: $\sigma = \sqrt{\sigma^2}$ **; geometric series is** $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ *if* $|x|<1$**, power:** $\sum_{n=0}^{N} x^n = \frac{1-x^{N+1}}{1-x}$ *if* $x \neq 1$

[empirical distribution's] sample mean: $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ **; series:** $\sum_{X=1}^{N} x = \frac{N(N+1)}{2}$**,** $\sum_{x=1}^{N} x^2 = \frac{n(n+1)(2n+1)}{6}$

[empirical distribution's] variance: $v = \sum_{i=1}^{n} (x_i - \bar{x})^2 (\frac{1}{n}) = \frac{1}{n}\sum_{i=1}^{n} (x_i - \bar{x})^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{x}^2$

[empirical distribution's] sample variance: $s^2 = \frac{1}{n-1}\sum_{i=1}^{n} (x_i - \bar{x})^2$

§ 2.1 Properties of Probability

P(A)=1-P(A'); mutually exlcusive when $A \cap B = empty\ set$**, so** $P(A \cup B) = P(A) + P(B)$
1=P(A)+P(A')

$P(A \cup B) = P(A) + P(B) - P(A \cap B), P(A \cup B) = P(A) + P(A' \cap B), P(A' \cap B) = P(B) - P(A \cap B)$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cup C) + P(A \cap B \cap C)$

$P(A) = \frac{h}{m} = \frac{N(A)}{N(S)}$ **; Q:** divide a line segment into two parts, prob. one part is at least twice as long as another, is 2/3

§ 2.2 Methods of Enumeration

Ordered w/o Replacement (permutation of n objects taken r at a time): ${}_nP_r = \frac{n!}{(n-r)!}$ (3-diff. color flags out of 4; 4 paintings in positions 1,..4 out of 9;)

Ordered w/ Replacement: n^r (combos of lock, pizza topings)

Unordered w/o Replacement (combination of n objects taken r at a time): ${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$ (world series games, cards, diff. kinds of candy – selected at random)

Unordered w/ Replacement: $\frac{(n-1+r)!}{r!(n-1)!}$

§ 2.3 Conditional Probability

…of event A given B occurred: $P(A|B) = \frac{P(A \cap B)}{P(B)}, if\ \ P(B) > 0$

probability that two events both occur (multiplication rule): $P(A \cap B) = P(A)P(B|A), P(B \cap A) = P(B)P(A|B)$

$P(A \cap B \cap C) = P(A)P(B|A)P(C|A \cap B)$

Q: Prob. of drawing two hearts: $P(A \cap B) = P(A)P(B|A) = \frac{13}{52}\frac{12}{51}$ (prob. of one, times prob. of drawing second given 51 cards left); $1 - P((A \cup B)') = P(A \cup B)$;

prob. of 3 kings in 13-card hand w/ at least 2 kings: A=3 or 4 Ks, B=2, 3, or 4 Ks,

$P(A|B) = \frac{N(A)}{N(B)} = \frac{\binom{4}{3}\binom{48}{10} + \binom{4}{4}\binom{48}{9}}{\binom{4}{2}\binom{48}{11} + \binom{4}{3}\binom{48}{10} + \binom{4}{4}\binom{48}{9}}$;

dice roll – list all outcomes; 6 decks, draw 6 cards, all different if P(A)= $\frac{52}{52}\frac{51}{52}\frac{50}{52}\frac{49}{52}\frac{48}{52}\frac{47}{52}$; prob they match is 1-P(A)

§ 2.4 Independent Events

…iff $P(A \cap B) = P(A)P(B)$

Also, these are independent: A and B', A' and B, A' and B'

A, B, C are mutually independent iff: pairwise independent

($P(A \cap B) = P(A)P(B), P(A \cap C) = P(A)P(C), P(B \cap C) = P(B)P(C)$ **) and** $P(A \cap B \cap C) = P(A)P(B)P(C)$

Also, these are independent: A and (BxC), A and (BuC), A' and (B'xC'), A' and B' and C' mut. indep.

Q: $P(A \cup B) = P(A) + P(B) - P(A)P(B)$ if indep.; 3 players kick football: prob. exactly one makes it is $P(A1 \cap A2' \cap A3') + P(A1' \cap A2 \cap A3') + P(A1' \cap A2' \cap A3)$ (and so on); coin tossed n times: prob. of any sequence is (1/2)^n, 3 heads of n is (n choose 3)*(1/2)^n

§ 3.1 Random Variables of the Discrete Type

r.v. X(s)=x; space of X={x:X(s)=x, sϵS}

p.m.f.: P(X=x)=f(x) such that f(x)>0, sum of all f(x)=1.

Hypergeometric distribution: $f(x) = \frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N}{n}}, x \le n, x \le N_1, n-x \le N_2$**,** $\mu = n\left(\frac{N_1}{N}\right), \sigma^2 = n\left(\frac{N_1}{N}\right)\left(\frac{N2}{N}\right)\left(\frac{N-n}{N-1}\right)$

(selecting r objects at random without replacement from a set composed of two types of objects)

Q: determine c such that f(x)=x*c was p.m.f., x=1,2…10: set 1=f(1)+..+f(10)=1c+2c+..+10c=55c, so c=1/55; rel. freq: N(X)/n; hypergeo-- defective items – exactly 1: P(X=1), at most 1: P(X=0)+P(X=1)

§ 3.2 Mathematical Expectation

…is $E[u(X)] = \sum_{x \in S} u(x)f(x)$**, mean is** $\mu = E(X)$ **too,** $\sigma^2 = E[(X-\mu)^2] = E(X^2) - \mu^2 = Var(X)$

if c is a constant, E(c)=c; c – constant and u – function, E[c u(X)]=cE[u(X)]; E[c1u1(X)+c2u2(X)] = c1E[u1(X)]+c2E[u2(X)]

Q: random integer from first 10: f(x)=1/10, E[X(11-X)]=11E[X]-E[X^2]; use series on pg. 1 to calculate E(X); lotto tickets: $0.50 each, 12,006 drawn out of 3Mil, 12,000-$25, 4-$10,000, 1-$50,000, 1-$200,000: E[X]=(12000*25+..+1*200000)/3Mil; value for us: E[X]-$0.50; E[X] is DNE when series does not converge;

skewness is $\frac{E[(X-\mu)^3]}{\sigma^3}$, top part is: $E[(X-\mu)^3] = E[X^3] - 3\mu E[X^2] + 2\mu^3$

§ 3.3 Bernoullis Trials and the Binomial Distribution

Bern. Trials: two different outcomes (yes/no) each experiment; p=prob. of success, q=1-p – of failure

Bernoulli Distribution: $f(x) = p^x(1-p)^{1-x}, x = 0,1$**,** $M(t) = 1 - p + pe^t$**,** $\mu = p, \sigma^2 = p(1-p)$**, x=0,1**

Binomial Distribution: $f(x) = \frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}, x = 0,1..,n$**,** $M(t) = (1-p+pe^t)^n$**,** $\mu = np, \sigma^2 = np(1-p)$**, X=# of success in n Bernoulli trials; b(n, p)**

Q: draw two diff. color balls: 7Red, 11White, X=1 if red, X=0 if white => Bernoulli and p=7/18; test w/ 6 Qs, 5 poss. anwers: P1(correct on #1 and #4)=P(C,I,I,C,I,I)=(1/5)^2*(4/5)^4; P2(correct on 2 Qs)=P1*(6 choose 2); when ask "what's expected value, plug in that number into E[#]);

binomial: $P(X \ge 5) = 1 - P(X \le 4)$; prob. of winning at least one prize in b(# of tickets bought, prob.) lotto is $P(X \ge 1) = 1 - P(X = 0) = 1 - \textit{use formula for } f(x)$

§ 3.4 The Moment-Generating Function

m.g.f: $M(t) = E(e^{tX}) = \sum_{x \in S} e^{tX} f(x)$**, -h<t<h,** $M'(t) = \sum_{x \in S} xe^{tX} f(x)$**,** $M''(t) = \sum_{x \in S} x^2 e^{tX} f(x)$**, M'(0)=E(X), M''(0)=E(X^2)**

$(1-z)^{-1} = 1 + z + z^2 + z^3 + ..., -1 < z < 1$**,** $\sigma^2 = M''(0) - \mu$

Negative Binomial Distribution: $f(x)=\binom{x-1}{r-1}p^r(1-p)^{x-r}, x=r,r+1,...,$ $M(t)=\frac{(pe^t)^r}{[1-(1-p)e^t]^r}, t>-\ln(1-p),$ $\mu=r(\frac{1}{p}), \sigma^2=\frac{r(1-p)}{p^2}$, **do Bern. trials until r successes, X=# of trial when success observed**

Geometric Distribution: $f(x)=(1-p)^{x-1}p$, $M(t)=\frac{pe^t}{1-(1-p)e^t}, t<-\ln(1-p)$, $\mu=\frac{1}{p}, \sigma^2=\frac{1-p}{p^2}$, **X=# of trials to obtain 1st success in seq. of Bern. trials**

Uniform Distribution: $f(x)=\frac{1}{m}, x=1,2,..,m$, $\mu=\frac{m+1}{2}, \sigma^2=\frac{m^2-1}{12}$

Q: when asked to find mean&etc knowing M(t), check to see which distr. M(t) is for ease; when we have b(n,p) with p>0.5, set Y is b(n,1-p), P(X<=x)=1-P(Y<=x);
Geo: people's birthdays (p=1/365), P(X>400)=(1-p)^400; P(X<300)=P(X<=299)=1-(1-p)^299;

$$P(X>k+j\,|\,X>k)=\frac{P((X>k+j)\cap(X>k))}{P(X>k)}=\frac{P(X>k+j)}{P(X>k)}=\frac{q^{k+i}}{q^k}=q^j=P(X>j);$$

§ 3.5 *The Poisson Distribution*

prob. of one change in interval length h is λh=λ(1/n); $f(x)=\frac{\lambda^x e^{-\lambda}}{x!}$, $M(t)=e^{\lambda(e^t-1)}$, $\mu=\lambda=\sigma^2$, **# of events occuring in a unit interval, events are occuring randomly at a mean rate of λ per unit interval**

Q: P(2<=X<=5)=P(X<=5)-P(X<=1); P(X>=3)=1-P(X<=2), P(X=2) =f(2); 11 cust/hr, P(X>10)=1-P(X<=10) @ mean=11; exp. mean=(flaws per unit)*length;

§ 4.1 Random Variables of Continious Type

Prob. dens. funct. (p.d.f.): f(x) satisfies: prob. of event $X\in A$ **is** $P(X\in A)=\int_A f(x)dx$ **on S, 0 elsewhere.**

[Cumilitive[distribution function (c.d.f.): $F(x)=P(X\le x)=\int_{-\infty}^{x} f(t)dt$, **F'(x)=f(x)=p.d.f.**

P(X=b)=0 and P(a<X<b)=…any sign <+=…=F(b)-F(a).

$\mu=E(X)=\int_{-\infty}^{\infty} xf(x)dx$, $\sigma^2=Var(X)=E[(X-\mu)^2]=\int_{-\infty}^{\infty}(x-\mu)^2 f(x)dx$, $M(t)=\int_{-\infty}^{\infty} e^{tx} f(x)dx, -h<t<h$

π_p **- (100p)th percentile (area under f(x) to the left of** π_p **is p):** $p=\int_{-\infty}^{\pi_p} f(x)dx=F(\pi_p)$

Q: to find Dist. Func, take integral of f(x); if $f(x)=\begin{cases}2(1-x), 0\le x\le 1\\ 0, otherwise\end{cases}$, then $F(x)=\{0, x\le 0; 1-(1-x)^2, 0<x<1; 1, x\ge 1\}$; find c such that f(x) was p.d.f., set 1=integral of f(x) from 0 to c; we usually replace minus-infiniti with lower limit

§ 4.2 *The Uniform and Exponential Distributions*

Uniform distribution: F(x)=(x-a)/(b-a) on a≤x<b**, 0 on x<a, 1 on x≥b, p.d.f: f(x)=1/(b-a), a≤x≤b; say, X is U(a,b)**

$\mu=\frac{a+b}{2}$, $\sigma^2=\frac{(b-a)^2}{12}$, $M(t)=\begin{cases}\frac{e^{tb}-e^{ta}}{t(b-a)}, t\ne 0; 1, t=0\end{cases}$ **(select a point at random from [a,b])**

Exponential distribution: **waiting time for 1st arrival when observing Poisson process with mean rate of change(arrivals)** $\lambda=1/\theta$**:** $f(x)=\frac{1}{\theta}e^{-x/\theta}, 0\le x\le\infty$, $M(t)=\frac{1}{1-\theta t}, t<1/\theta$, $\mu=\theta$, $\sigma^2=\theta^2$, $M'(t)=\frac{\theta}{(1-\theta t)^2}$,

$M''(t)=\frac{2\theta^2}{(1-\theta t)^3}$, $F(x)=\{1-e^{-x/\theta} on\, 0\le x<\infty, 0\, on\, -\infty\le x<0$, **median m:** $m=\theta\ln(2)$, $P(X>x)=e^{-x/\theta}$

Q: Uni: P(X>=8)=1-P(X<=8)=1-F(8) (see 1st line); P(2<=X<=8)=F(8)-F(2); customer arrives in a certain window of time; **EXP:** P(10<X<30)=F(30)-F(10), P(X>=30)=1-F(30), P(X>40|X>10)=P(X>30)=1-F(30); waiting time until 1st call arrives lamda=calls/min=poisson mean/length;

§ 4.3 *The Gamma and Chi-Square Distributions*

Gamma function: $\Gamma(t)=\int_0^\infty y^{t-1}e^{-y}dy, 0<t$, $\Gamma(n)=(n-1)!$

Gamma distribution: **like exponential, but waiting until αth arrival.** $f(x)=\dfrac{x^{\alpha-1}e^{-x/\theta}}{\Gamma(\alpha)\theta^\alpha}, 0<x<\infty$,

$M(t)=\dfrac{1}{(1-\theta t)^\alpha}, t<1/\theta$, $\mu=\alpha\theta$, $\sigma^2=\alpha\theta^2$, $F(w)=1-\sum_{k=0}^{\alpha-1}\dfrac{(\lambda w)^k e^{-\lambda w}}{k!}$

Chi-square distribution w/ r degrees of freedom: **gamma with θ=2, α=1/2; sum of squares of r independent N(0,1) variables. X is** $\chi^2(r)$. $f(x)=\dfrac{x^{r/2-1}e^{-x/2}}{\Gamma(r/2)2^{r/2}}, 0\le x<\infty, M(t)=\dfrac{1}{(1-2t)^{r/2}}, t<1/2$, $\mu=r$, $\sigma^2=2r$,

$F(x)=\int_0^x \dfrac{w^{r/2-1}e^{-w/2}dw}{\Gamma(r/2)2^{r/2}}$, **r=1, 2…**

Q: Gamma – until nth call arrives; P(X<=5)=F(5)=(use F(5)); P(X>=35)- use intergral from 35 to + inf; if <35, use –inf to 35 or 1-P(X<=35); $(7/120)^2\int_{35}^{\infty} x\exp[-7x/120] = (7/120)[-x\exp[-7x/120]]_{35}^{\infty} - [\exp[-7x/120]]_{35}^{\infty}$; Chi-square – use table; if 15 observ. , 4 degr. Freedom, find prob. that at most 3 of 15 obs. Exceed 7.779. P(X<=7.779)=0.9 (table IV), so we have b(15, 0.9), add up for P(X=0)+P(X=1)+P(X=2)+P(X=3)

§ 4.4 *The Normal Distribution*

Errors in measurements, heights of children, breaking strengths; $-\infty<\mu<\infty$, $0<\sigma$, $f(x)=\dfrac{\exp(-(x-u)^2/2\sigma^2)}{\sigma\sqrt{2\pi}}$,

$M(t)=\exp(\mu t+\sigma^2t^2/2)$, $\mu=E(X)=\mu$, $\sigma^2=Var(X)=\sigma^2$, **X is N(**μ,σ^2**)**

If Z is N(0,1), Z has ***standard norm. dist.*****:** $\Phi(z)=P(Z\le z)=\int_{-\infty}^z \dfrac{e^{-w2/2}dw}{\sqrt{2\pi}}$, $\Phi(-z)=1-\Phi(z)$

$P(a\le X\le b)=P(\dfrac{a-\mu}{\sigma}\le\dfrac{X-\mu}{\sigma}\le\dfrac{b-\mu}{\sigma})=\Phi(\dfrac{b-\mu}{\sigma})-\Phi(\dfrac{a-\mu}{\sigma})$ **b/c** $(X-\mu)/\sigma$ **is N(0,1)**

If X is N(μ,σ^2**), then V=**$(X-\mu)^2/\sigma^2$**=Z²** **is** $\chi^2(1)$

§ 5.1 Distributions of Two Random Variables

f(x, y)=P(X=x, Y=y); joint p.m.f. satisfies: $\sum\sum_{(x,y)\in S} f(x,y)=1$ **and** $P[(X,Y)\in A]=\sum\sum_{(x,y)\in A} f(x,y)$ **(similar for pdf, intergrals from – to + infinity); Marginal p.m.f. of X:** $f_1(x)=\sum_y f(x,y)=P(X=x), x\in S_1$ **(same for Y); X and Y independent iff P(X=x, Y=y)=P(X=x)P(Y=y) or f(x,y)=f1(x)f2(y), otherwise – dependent (when support S is not "rectangular"). Expected value of** $u(X_1,X_2)$**:** $E[u(X_1,X_2)]=\sum\sum_{(x1,x2)\in S} u(x1,x2)f(x,y)$ **; mean of Xi is** $E[u_1(X_1,X_2)]=E(X_i)=\mu_i$**, if u1=Xi; variance of Xi is** $E[u_2(X_1,X_2)]=E[(X_i-\mu_i)^2]=\sigma_i^2$**, i=1,2**

Hypergeometric distribution for 3: $f(x1,x2)=\dfrac{\binom{N_1}{x1}\binom{N_2}{x2}\binom{N-N_1-N_2}{n-x1-x2}}{\binom{N}{n}}$ **(marginal pmf are regular hypergeo. dist.)**

Trinomial disrtibution: $f(x_1,x_2)=\dfrac{n!p_1^{x1}p_2^{x2}(1-p_1-p_2)^{n-x1-x2}}{x_1!x_2!(n-x_1-x_2)!}$, **X1 is b(n,p1), X2 is b(n, p2), X1 & X2 dependent**

Q: Marginals: of x: (x+10)/3, of y: (10+y)/3; P(X>Y)=(sum of all scenarios where X>Y); same for P(X+Y=3)=P(X=1,Y=2)+P(X=2, Y=1); sample support for hypergeo (cards): S={(X1,X2) | 0<=x1+x2<=13, 0<=x1, x2<=13}; trinomial: S={(x,y)| 0<=x, y<=7, x+y<=7}—triangular; trinomial P(X<=11): use marginal pmf for X.

§ 5.2 The Correlation Coefficient

Covariance of X1 and X2: $E[u_3(X_1,X_2)]=E[(X_1-\mu_1)(X_2-\mu_2)]=\sigma_{12}=Cov(X_1,X_2)$ **if** $u_3(X_1,X_2)=(X_1-\mu_1)(X_2-\mu_2)$**; Correlation coefficient of X1 and X2:** $\rho=\frac{Cov(X_1,X_2)}{\sigma_1\sigma_2}=\frac{\sigma_{12}}{\sigma_1\sigma_2}$ **if std. devs. >0**

$\boxed{\mu_1=E(X_1)=\sum_{x1}\sum_{x2}x_1f(x_1,x_2)=\sum_{x1}x_1\left[\sum_{x2}f(x_1,x_2)\right]=\sum_{x1}x_1f_1(x_1)}$**; Covariance:** $\boxed{Cov(X_1,X_2)=E(X_1X_2)-\mu_1\mu_2}$

Least-squares regression line: slope $\rho=\frac{\sum_S(x-\mu_x)(y-\mu_y)f(x,y)}{\sigma_x\sigma_y}$**, then line is** $y=\mu_y+\rho\frac{\sigma_y}{\sigma_x}(x-\mu_x)$**, exp. value of square is** $K(b)=E\{[(Y-\mu_y)-b(X-\mu_x)]^2\}$**, its min:** $K(\rho\frac{\sigma_y}{\sigma_x})=\sigma_y^2(1-\rho^2)$**; corr. coeff of two indep. variables is 0;**

Sample Correlation Coefficient for Emperical Distribution:

$r=\frac{\sum_{i=1}^{n}x_iy_i-n\overline{x}\overline{y}}{\sqrt{\sum_{i=1}^{n}x_i^2-n\overline{x}^2}\sqrt{\sum_{i=1}^{n}y_i^2-n\overline{y}^2}}$**, sample least squares regression line is** $\hat{y}=\overline{y}+r\frac{s_y}{s_x}(x-\overline{x})$

Q: Use boxed formula for means of x and y; $\sigma_x^2=\sum_x(x-\mu_x)^2f(x)=\sum_x x^2f(x)-\mu_x{}^2$, E(XY)=sum of all combinations of x and y and the function: =x1*y1*f(x1,y1)+x2*y1*f(x2,y1)+x1*y2*f(x1,y2)+x2*y2*f(x2,y2); trinomial dist – E(X) and E(Y): calculate f1(1), f1(2),…, f2(1), f2(2),…, then add to E(X) and E(Y) respectively, Var(X)=0^2*f1(0)+..+n^2*fn(n) –mean-squared; to get corr. Coef. calc means of x and y, calculate E(XY), then Cov(X,Y), then plug in the numbers

§ 5.3 Conditional Distributions

conditional p.m.f. of X, given Y=y, is $g(x\mid y)=\frac{f(x,y)}{f_2(y)}$ **provided f2(y)>0; of Y, given X=x, is** $h(y\mid x)=\frac{f(x,y)}{f_1(x)}$**; cond. prob.:** $P(a<Y<b\mid X=x)=\sum_{\{y:a<y<b\}}h(y\mid x)$**; cond. expectation:** $E[u(Y)\mid X=x]=\sum_y u(y)h(y\mid x)$**; cond. mean of Y, given X=x,** $\mu_{Y|x}=E(Y\mid x)=\sum_y yh(y\mid x)$**; cond. var. of Y,** $\sigma_{Y|x}^2=E\{[Y-E(Y\mid x)]^2\mid x\}=\sum_y[y-E(Y\mid x)]^2h(y\mid x)$**, also** $\sigma_{Y|x}^2=E(Y^2\mid x)-[E(Y\mid x)]^2$**; if E(Y|x) is linear, then it is** $E(Y\mid x)=\mu_y+\rho\frac{\sigma_y}{\sigma_x}(x-\mu_x)$ **(same as best-fit line)**

For Trinomial: X and Y have marginal binomial distributions b(n, p1) and b(n, p2); thus

$h(y\mid x)=\frac{f(x,y)}{f(x)}=\frac{(n-x)!}{y!(n-x-y)!}\left(\frac{p_2}{1-p_1}\right)^y\left(\frac{p_3}{1-p_1}\right)^{n-x-y}$**, thus conditional p.m.f. of Y, given X=x, is binomial** $\boxed{b\left[n-x,\frac{p_2}{1-p_1}\right]}$**, cond. means** $\boxed{E(Y\mid x)=(n-x)\frac{p_2}{1-p_1}}$ **and** $E(X\mid y)=(n-y)\frac{p_1}{1-p_2}$**;** $\rho=-\sqrt{\frac{p_1p_2}{(1-p_1)(1-p_2)}}$

Q: weight of soap observed n times is b(n, p), so are eyes of fly; use E(X)=np, Var(X)=np(1-p) formulas; how distributed conditionally – find h(y|3) or g(3|x) in terms of y or x respectivelly, if binomial, use formula; to find E(X^2-3XY+3Y^2), find E(X), then E(Y), Var(X), Var(Y), E(X^2)=E(X)^2+Var(X), calculate corr. coef. using formula in section, calculate E(XY), plug all #s in. "Let X have a… distr. and let condit. distr. of Y, given X=x, be…", then joint p.d.f. is f(x,y)=h(y|x)*f1(x), sample $E(X \mid y) = \int_{\sqrt{y}}^{2} g(x \mid y)dx$

(draw pics here)

§ 6.1 Independent Random Variables

P(X1=x1 and X2=x2)=P(X1=x1)P(X2=x2)=f1(x1)f2(x2)=joint p.m.f.

Expected value of Y=u(X1, X2): $E[u(X_1, X_2)] = \sum_{x2 \in S2} \sum_{x1 \in S1} u(x_1, x_2) f_1(x_1) f_2(x_2) = \sum_{y \in S} y g(y)$**, g(y) – p.m.f. of Y.**

If X1 and X2 indep., exp. value of product u1(X1)u2(X2) is product of exp. Values of u1(X1) and u2(X2) and $E[(X_1 - \mu_1)(X_2 - \mu_2)] = E(X_1 - \mu_1)E(X_2 - \mu_2) = 0$

If r.v.'s 1..n have same p.m.f., then we say "random sample size n=? from a distribution w/ p.m.f. f(x)" – joint p.m.f. is f1(x)f2(x).

Q: P(X1=a,X2=b)=f1(a)f2(b); if P(X1+X2=a), sum the possible combos, i.e. P(X1=2, X2=5)+P(X1=3,X2=4); if ranged and pdf, do intergrals each over ranges; if $E(X_1^2 X_2^3) = \ldots = \int_{0toA} x_1^{\,2} f_1(x_1)dx_1 \int_{0toB} x_2^{\,3} f_2(x_2)dx_2$; $\sigma^2 = E(X^2) - E(X)$; if $P(\max X_i < a) = P(X_1 < a)\ldots P(X_n > a)$; $E(X_i^{\,2}) = \mu^2 + \sigma^2$

§ 6.2 Distributions of Sums of Independent Random Variables

Convolution formula: $g(y) = P(Y = y) = \sum_{k=1}^{y-1} f(k) f(y-k)$

Thm: X1, X2,…Xn – indep. r. v.s w/ joint pmf f1(x1)*..*fn(xn); let Y=u(X1, X2,..,Xn) have pmf g(y), then $E(Y) = \sum_{y} y g(y) = \sum_{x1} \sum_{x2} \ldots \sum_{xn} u(x_1, x_2, \ldots, x_n) f_1(x_1) f_2(x_2) \ldots f_n(x_n)$ **(for cont. type, integrals replace Sum)**

Thm: If X1, X2,…Xn – indep. r. v.s then $E[u_1(X_1)u_2(X_2)\ldots u_n(X_n)] = E[u_1(X_1)]\ldots E[u_n(X_n)]$

Thm: Mean and Var. of $Y = \sum_{i=1}^{n} a_i X_i$ **where *a*1,..*a*n are real constants, are** $\mu_Y = \sum_{i=1}^{n} a_i \mu_i$ **and** $\sigma_Y^2 = \sum_{i=1}^{n} a_i \sigma_i^2$

Sample mean: $\overline{X} = \dfrac{X_1 + X_2 + \ldots + X_n}{n}$ **(then *a*i=1/n)**

Thm: If X1, X2…Xn – indep. r. v.s with $M_{x_i}(t)$**, then m.g.f. of** $Y = \sum_{i=1}^{n} a_i X_i$ **is** $M_Y(t) = \prod_{i=1}^{n} M_{Xi}(a_i t)$

Cor: If X1, X2.., Xn observations of a random sample with m.g.f. M(t), then (a) the m.g.f. of $Y = \sum_{i=1}^{n} X_i$ **is** $M_Y(t) = [M(t)]^n$**; (b) m.g.f. of** $\overline{X} = \sum_{i=1}^{n} (1/n) X_i$ **is** $M_{\overline{X}}(t) = [M(\frac{t}{n})]^n$

Q: find mean E(Xi) then E(Xi^2), plug in; Y=-2X1+X2, $\mu_1 = 3, \mu_2 = 7, \sigma_1 = 9, \sigma_2 = 25$, then E(Y)=-2*3+7; Var(Y)=(-2)^2*9+(1)^2*25; to show which distr., compute M(t) using theorem.

§ 6.3 Random Functions Associated with Normal Distributions

Thm: If X1, X2, ..Xn are observations of a random sample of size n from the normal distribution N(μ, σ^2), then the distribution of sample mean $\bar{X} = \sum_{i=1}^{n} (1/n) X_i$ is N($\mu, \sigma^2/n$)

Thm: Let indep. distributions X1, X2…Xn be $\chi^2(r_i)$, then Y=X1+…+Xn is $\chi^2(r_1 + r_2 + ... + r_n)$

Thm: Let indep. Z1, Z2…Zn have stand. norm. dist. – N(0,1). Then W=Z1^2+..+Zn^2 is $\chi^2(n)$

Thm: If X1, X2, … Xn are observations from a random sample of size n from normal distr. N(μ, σ^2), $\bar{X} = \sum_{i=1}^{n} (1/n) X_i$ and $S^2 = (1/n-1)\sum_{i=1}^{n} (X_i - \bar{X})^2$, then (a) $\bar{X}$ and S^2 are independent;

(b) $\frac{(n-1)S^2}{\sigma^2} = \frac{\sum_{i=1}^{n}(X - \bar{X})^2}{\sigma^2}$ is $\chi^2(n-1)$. NOTE: sampling from norm. distr. $U = \sum_{i=1}^{n} \frac{(X_i - \mu)^2}{\sigma^2}$ is $\chi^2(n)$ and $W = \sum_{i=1}^{n} \frac{(X_i - \bar{X})^2}{\sigma^2}$ is $\chi^2(n-1)$; Thm: If X1, X2…Xn are n mutually indep. norm. var., then $Y = \sum_{i=1}^{n} c_i X_i$ has norm. dist. $N(\sum_{i=1}^{n} c_i \mu_i, \sum_{i=1}^{n} c_i^2 \sigma_i^2)$. Note: if Y=X1-X2, then Y is N($\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2$)

Q: figure out mean and var and use $P(a \le X \le b) = P(\frac{a-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{b-\mu}{\sigma}) = \Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})$; **Soap boxes:** X – weight of box, X is N(6.05, 0.0004), 9 boxes selected @ random, how many < 6.0171? A: W. W is b (9, 0.05) and $P(W \le 2) = \binom{9}{0}(0.95)^9 + \binom{9}{1}(0.05)(0.95)^8 + \binom{9}{2}(0.05)^2(0.95)^7$; Zs, Ws, random sample – do chi-square thm; when chi-square, find which of two forms it is, look up values in table; "let S^2 be sample variance of 9 weights" means it's $\chi^2(8)$; find P(X1>X2): set Y=X1-X2, so P(Y>0), use **Note** to get values for norm. dist., then do as usual.

§ 6.4 The Central Limit Theorem

Mean $\bar{X}$ of a random sample of size n from a distr. with mean μ and var. $\sigma^2 > 0$ is a rand. var. with: $E(\bar{X}) = \mu$ and $Var(\bar{X}) = \frac{\sigma^2}{2}$; CLThm: If $\bar{X}$ is the mean of a random sample X1, X2…Xn, of size n from a disribution with a finite mean μ and a finite variance $\sigma^2 > 0$, the the distribution of $W = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{\sum_{i}^{n} Xi - n\mu}{\sqrt{n}\,\sigma}$ is N(0,1) in the limit as $n \to \infty$. So, $P(W \le w) \approx \Phi(w)$; and $\bar{X}$ is appr. $N(\mu, \sigma^2/n)$ and Y is appr. $N(n\mu, n\sigma^2)$

Q: find mean E(Xi) and E(Xi^2); use mean or sum formula as appropriate; Y=X1+…+Xn is $\chi^2(n)$**;**