High-dimensional data analysis seminar

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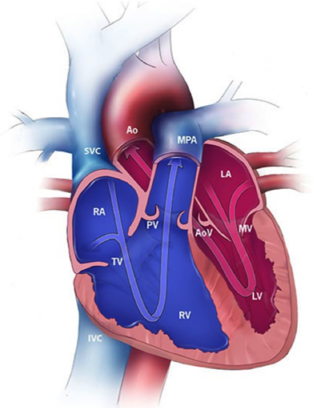
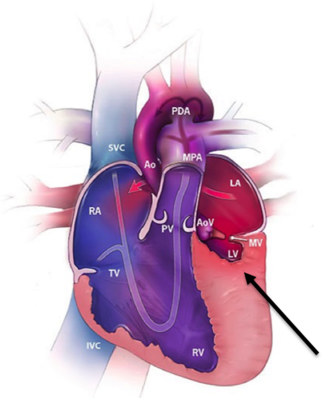
Outline for the talk

- Clinical motivation.
- Data sets and computational models.
- A registration problem (w/ Lior, Weigand).
- A modeling problem (w/ LaPole, Olufsen, Lior, Weigand).

Hypoplastic left heart syndrome



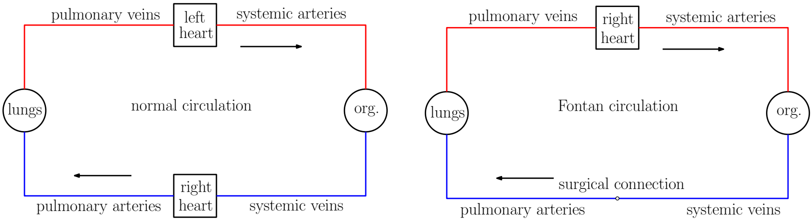
Normal Heart



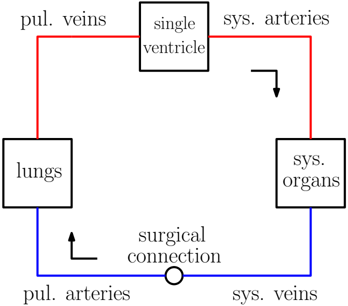
HLHS Heart

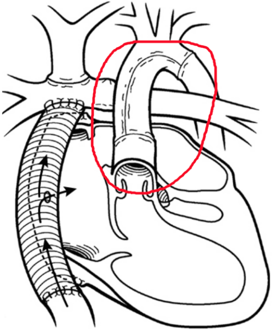
(Figure adapted from `http://www.cdc.gov/ncbddd/heartdefects/hlhs.html`.)

Normal and Fontan circulations



The Fontan circulation





effects of aortic reconstruction on flow and perfusion?

Multi-institutional registry data

Multi-institutional registry data

$>$ 4000 unique patients...

- 4D flow data: blood velocity field over a cardiac cycle.
- MRA data: high resolution imaging of anatomy.
- Demographic information.
- Type of surgical technique.
- Clinical outcomes: survival rate, time-to-transplant, complications, etc...

Imaging data

- 4D Flow (velocity): can be used to quantify hemodynamics.

  poor spatial resolution.

- MRA (geometry): provides "gold standard" anatomy.

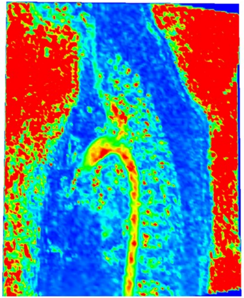
  no hemodynamic information.

- Both types can be used in computer simulations.

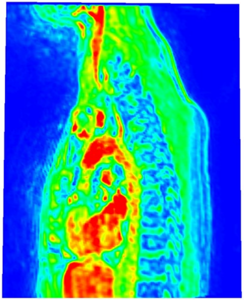
  4D Flow $\rightarrow$ to set boundary conditions and to calibrate/validate.

  MRA $\rightarrow$ to construct the computational domain.

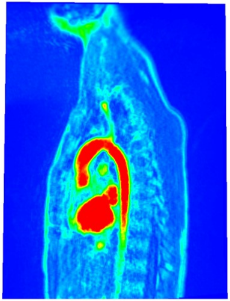
Visualization of data



velocity mag. (4D Flow)



magnitude (4D Flow)

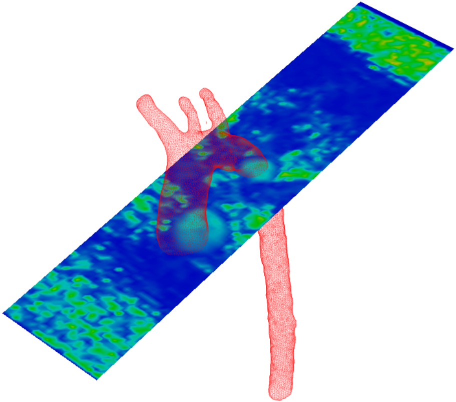


density (MRA)

Goal: Simultaneously use MRA and 4D Flow data.

Misalignment of MRA and 4D Flow data

Main sources of misalignment: patient shifting between scans, or scans taken at separate times.

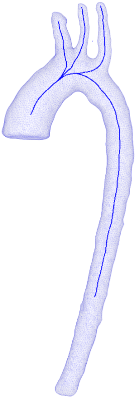


Registration of MRA and 4D Flow data

Two key hypotheses:

1. Vessel centerlines can be extracted from noisy data.
2. Registration determined by the alignment of centerlines.

4D Flow data



MRA data

Proposed registration procedure

1. Create centerlines Γ_{fixed} from 4D Flow and Γ_{floating} from MRA.
2. Compute a rigid transformation T_{rigid} so that:

$$T_{\text{rigid}}(\Gamma_{\text{floating}}) \approx \Gamma_{\text{fixed}}.$$

3. Compute a nonrigid transformation T_{nonrigid} so that:

$$T_{\text{nonrigid}}\left(T_{\text{rigid}}(\Gamma_{\text{floating}})\right) \approx \Gamma_{\text{fixed}}.$$

4. Extend $d(u) = T_{\text{nonrigid}}(T_{\text{rigid}}(u)) - u$, $u \in \Gamma_{\text{floating}}$, to $\mathbb{R}^3$.
5. Apply extended displacement field to MRA segmentation.

Lior et al., *arXiv:2312.03116*, 2025.

Rigid transformation T_{rigid}

An association of points along each centerline is defined:

For each $u \in \Gamma_{\text{floating}}$, compute the "closest" $\tilde{u} \in \Gamma_{\text{fixed}}$.

A rigid transformation is the solution to the following:

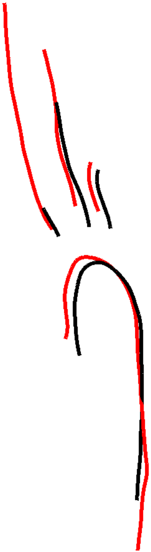
$$\min_{\Omega} \|\Omega P_{\text{floating}} - P_{\text{fixed}}\|_F$$
$$s.t. \quad \Omega^T \Omega = I.$$

This is called the "Procrustes" problem.

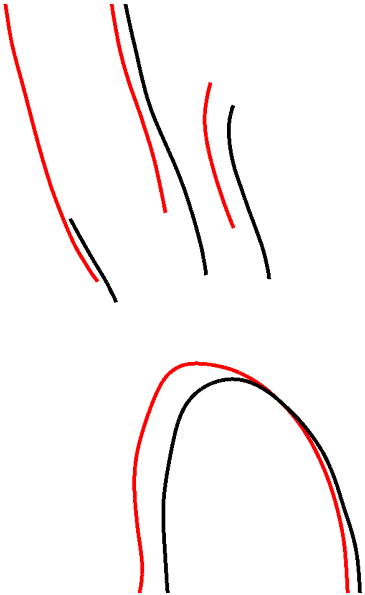
This procedure is applied iteratively until $\Omega \approx I$.

T_{rigid} is defined as the composition of all Ω's from all iterations.

Movie: Γ_{fixed} is **red** and Γ_{floating} is **black**.



Movie: Γ_{fixed} is **red** and Γ_{floating} is **black**.



Nonrigid transformation T_{nonrigid}

An association of points along each centerline is defined:

For each $u \in T_{\text{rigid}}(\Gamma_{\text{floating}})$, compute the "closest" $\tilde{u} \in \Gamma_{\text{fixed}}$.

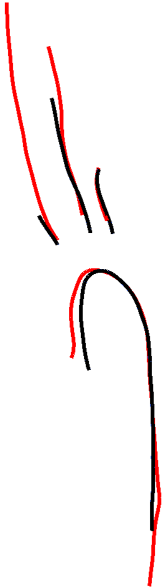
For a parameter $0 < \sigma < 1$, define:

$$\Psi(u) = \sigma u + (1 - \sigma)\tilde{u}.$$

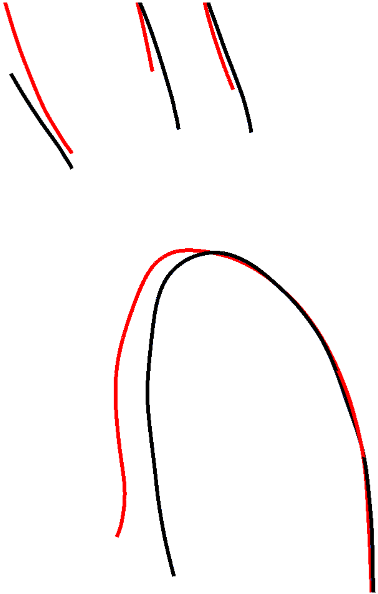
This procedure is applied iteratively until max distance between any two points is less than a tolerance.

T_{nonrigid} is defined as the composition of all Ψ's from all iterations.

Movie: Γ_{fixed} is **red**, $T_{\text{rigid}}(\Gamma_{\text{floating}})$ is **black**, and the intermediate is **blue**.



Movie: Γ_{fixed} is **red**, $T_{\text{rigid}}(\Gamma_{\text{floating}})$ is **black**, and the intermediate is **blue**.



Extension of centerline displacement

Given a displacement field defined on a centerline:

$$d(u) = T_{\text{nonrigid}}(T_{\text{rigid}}(u)) - u, \quad u \in \Gamma_{\text{floating}},$$

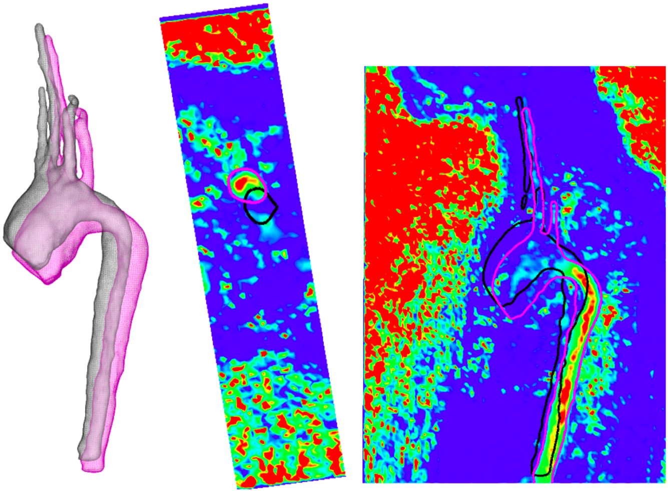
- Define a grid of B-splines on the imaged region. Denote

$$\begin{aligned} \alpha &= \text{degrees of freedom for the B-spline field,} \\ \Phi(\alpha) &= \text{B-spline field,} \\ \mathcal{I}[\Phi(\alpha)] &= \text{interpolation of B-spline field to centerline.} \end{aligned}$$

- Solve the following optimization problem:

$$\min_{\alpha} \|\mathcal{I}[\Phi(\alpha)] - d\|_2^2 + \gamma\|\alpha\|_2^2 + \beta\|L\alpha\|_2^2$$

where L is the graph Laplacian on the grid of B-splines.



What to do with the data?

Geometric analysis using the MRA segmentation:

- Calculate scalar features along the centerline and do "standard" statistical analysis of these quantities.
- Can we take functions of the vessel centerline directly into the analysis?
- Can we do more sophisticated shape analysis?

Analysis of 4D flow with registered MRA:

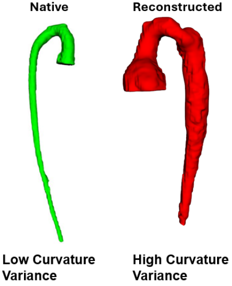
- Calculate features from velocity field.
- Correlate geometric and velocity derived features.

Translation of these results:

- Identify relationships between features and clinical outcomes.
- Can we use models to tell the surgeon what to do?

Preliminary geometric analysis

Feature	Mean p-value	Geometric Interpretation	Clinical Correlate
Logit Model 1:			
Minimum semi minor axes change	0.024	Tapering	Substrate for energy loss
Logit Model 2:			
Variance of curvature	0.057	Abrupt changes in centerline trajectory	Substrate for energy loss
Average cross-section diameter change	0.009	Tapering	



Computer modeling for Fontan

- Compartmental models: exercise tolerance, fenestration, ...

- Vessel network models: fenestration, aortic reconstruction, liver perfusion, uncertainty quantification ...

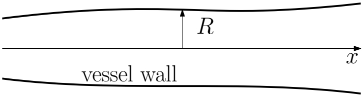
- CFD and FSI models: to be done ... nothing published yet...

A model for flow in a single vessel

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\alpha \frac{Q^2}{A}\right) + \frac{A}{\rho}\frac{\partial p}{\partial x} = -2\pi\nu R \frac{\partial u_x}{\partial r}\bigg|_{r=R}$$

$$p = p_0 + \psi(A; A_0, \beta)$$



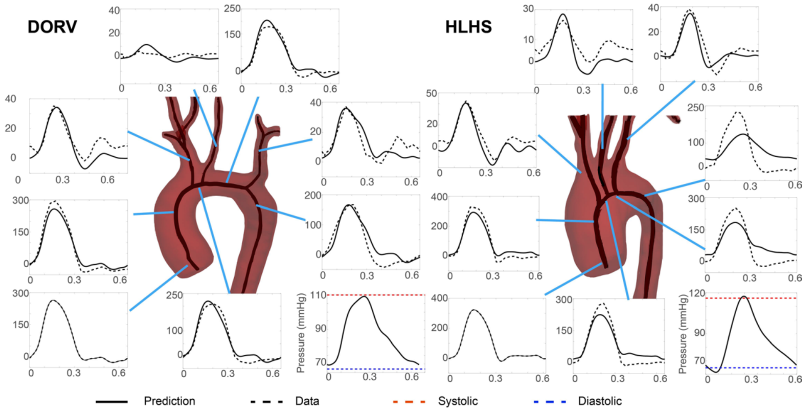
- $A = \pi R^2$ is the vessel cross–sectional area.
- $Q = AU$ is the momentum, where U is the average axial velocity.
- p is the pressure.

Canic and Kim, *Math Method Appl Sci*, 2003

Hughes, *PhD thesis, University of California, Berkeley*, 1974

Hughes and Lubliner, *Math Biosci*, 1973

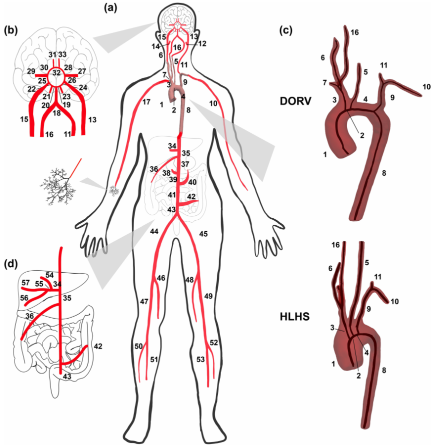
Calibrated models to 4Dflow and pressure data

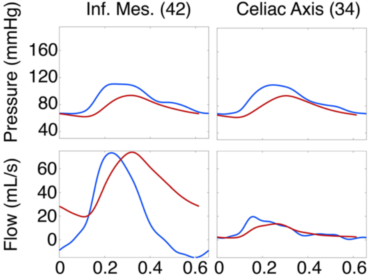


Taylor-LaPole, Colebank, Weigand, Olufsen, Puelz, *Biomech Model Mechan*, 2023

Taylor-LaPole, Paun, Lior, Weigand, Puelz, Olufsen, *J R Soc Interface*, 2025

Vessel network models to predict perfusion





blue = DORV, red = HLHS

Region	Rest		Exercise	
	DORV (%)	HLHS (%)	DORV (%)	HLHS (%)
Cerebral	8.6	7.3	4.8	5.2
Liver and gut	24.3	15.7	20.2	9.4
Lower body	25.7	48.0	40.0	60.8
Other	41.4	29.0	35.0	24.6

These values are shown as percentages of the stroke volume

Taylor-LaPole, Colebank, Weigand, Olufsen, Puelz, *Biomech Model Mechan*, 2023