

Homework 2

1. On the sides of a thin rod, heat exchanges takes place (obeying Newton's law of cooling) - flux proportional to temperature difference with a medium of constant temperature T_0 .

What is the equation satisfied by the temperature $U(x,t)$, neglecting its variation across the rod?

2. Derive the equation of one-dimensional diffusion in a medium that is moving along the x axis to the right at constant speed V .

3. If $f(x)$ is continuous and $|f(x)| \leq \frac{1}{|x|^3+1}$

for all x show that.

$$\iiint_{\text{all space}} \nabla \cdot f \, dx = 0$$

(Hint: Take D to be a large ball, apply the divergence theorem, and let the radius tend to $+\infty$)

4. If $\text{curl } v = 0$ in all of three-dimensional space, show that there exists a scalar function $\phi(x, y, z)$ such that $v = \nabla \phi$