Homework 3

1. Consider the problem

$$U''(x) + U'(x) = f(x)$$

$$U'(0) = U(0) = \frac{1}{2}[U'(L) + U(L)]$$

with $f(x)$ a given function

i) Is the solution unique? Explain

ii) Does a solution necessarily exists, or is there a condition that $f(x)$ must satisfy for existence? Explain

2. what are the types of the following equations?

a) $U_{xx} - U_{xy} + 2U_y + U_{yy} - 3U_{yx} + 4U = 0$

b) $9U_{xx} + 6U_{xy} + U_{yy} + U_x = 0$

3. Consider the equation: $3U_y + U_{xy} = 0$

i) what is its type?

ii) Find the general solution (Hint: Substitute $V = U_y$)

iii) With the auxiliary conditions $U(x,0) = e^{-3x}$ and $U_y(x,0) = 0$ does a solution exist? Is it unique?

4. Solve $u_{tt} = c^2 u_{xx}$, $u(x,0) = e^x$, $u_t(x,0) = \sin x$

5. Solve $u_{tt} = c^2 u_{xx}$, $u(x,0) = \log(1+x^2)$, $u_t(x,0) = 4+x$