Homework 5

Due 04/03/15

1. Solve the completely inhomogeneous diffusion problem on the half-line

$$\begin{cases} V_t - kV_{xx} = f(x,t) & \text{for } \begin{matrix} 0<x<\infty \\ 0<t<\infty \end{matrix} \\ V(0,t) = h(t), \quad V(x,0) = \phi(x) \end{cases}$$

by carrying out the substraction method begun in the text, Section 3.3.

2. Solve $\begin{cases} U_{tt} = 9U_{xx} & 0<x<L \\ U(x,0) = \cos x, \quad U_t(x,0) = 0 \\ U_x(0,t) = 0, \quad U(\frac{\pi}{2}, t) = 0 \end{cases}$

3. Solve

$$U_{tt} = c^2 U_{xx} + e^{ax}, \quad x \in \mathbb{R}$$

$$U(x,0) = 0, \quad U_t(x,0) = 0$$

4. Derive the formula for the inhomogeneous wave equation

$$(1)\quad \begin{cases} v_{tt} - c^2 v_{xx} = f(x,t) & x \in \mathbb{R} \\ v(x,0) = \phi(x), \quad v_t(x,0) = \psi(x) \end{cases}$$

as follows.

(a) Write it as the system

$$(2)\quad \begin{cases} v_t + c v_x = w \\ w_t - c w_x = f \end{cases}$$

(b) Solve the first equation above in (2) to get

$$v(x,t) = \int_0^t w(x - ct + cs, s)\, ds$$

(c) Similarly, solve the second equation for w in terms of f in (2)

(d) Substitute part (c) into part (b) and write as an iterated integral.