

Homework 6

Due 04/10/15

1. Solve

$$\begin{cases} U_{tt} = c^2 U_{xx} & \text{for } 0 < x < +\infty \\ U(0,t) = t^2, \quad U(x,0) = x, \quad U_t(x,0) = 0 \end{cases}$$

2. Consider waves in a resistant medium that satisfy the problem

$$\begin{cases} U_{tt} = c^2 U_{xx} - \eta U_t & \text{for } 0 < x < L \\ U(0,t) = 0 = U(L,t) \\ U(x,0) = \phi(x), \quad U_t(x,0) = \psi(x) \end{cases}$$

where $0 < \eta < \frac{2\pi c}{L}$. Write down the series expansion of the solution. Do the same for

$$\frac{2\pi c}{L} < \eta < \frac{4\pi c}{L}$$

3. A quantum - mechanical particle on the line with an infinite potential outside the interval $(0, L)$ ("particle in a box") is given by Schrodinger equation $U_t = iU_{xx}$ in $(0, L)$ with

Dirichlet conditions at the ends.

Separate variables and find ~~the~~ the series representation of the solution

4. a) Use the Fourier expansion to explain why the note produced by a violin string rises sharply by one octave when the string is clamped exactly at its midpoint

b) Explain why the note rises when the string is tightened