Homework 7

Due 04/17/15

1. Solve the diffusion problem

$$\begin{cases} U_t = k U_{xx} & x \in (0, L) \\ U(0,t) = U_x(L,t) = 0 \\ U(x,0) = \phi(x) = \sin \frac{\pi x}{2L} \end{cases}$$

. Give the solution for $\phi(x) = \sin \frac{\pi x}{2L}$

2. Consider diffusion inside an enclosed circular tube. Let its length be $2L$. Let x denote the arc length parameter where

$$-L \le x \le L$$

Then the concentration of the diffusing substance satisfies

$$(1) \begin{cases} U_t = k U_{xx} \quad \text{for} \quad -L \le x \le L \\ U(-L,t) = U(L,t) \quad \text{and} \quad U_x(-L,t) = U_x(L,t) \end{cases}$$

These are called periodic boundary conditions

Show that

$$U(x,t) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right) e^{-\frac{n^2 \pi^2 k t}{L^2}}$$

Solve problem (1) with

$$U(x,0) = \cos \frac{\pi x}{L} + \sin \frac{2\pi x}{L}$$