Final exam

MA 4335

Due May 13 - 2015,

Info: Matlab or any other software !!!

Each problem is ~~option~~ with 10 points. Total 10 problems. Matlab requirements are optional and earn extra credit, 2.5 points per good Matlab plot.

1. Solve the equations. fn $U(x,y)$

a) $3U_y + U_{xy} = 0$

b) $\sqrt{1-x^2}\,U_x + U_y = 0$ with $U(0,y) = y$.

Matlab: Plot the characteristic curves.

2. Determine the regions of the xy plane where the equation

$$(1+x)U_{xx} + 2xyU_{xy} - y^2U_{yy} = 0$$

is elliptic, hyperbolic or parabolic.

Matlab: sketch these regions.

3. Solve for $U(x,t)$

$$\begin{cases} U_{xx} + U_{xt} - 20U_{tt} = 0 & \text{in } \mathbb{R} \times (0,+\infty) \\ U(x,0) = \sin x \,, \; U_t(x,0) = 0 & \text{in } \mathbb{R} \end{cases}$$

Matlab: Plot the solution.

4. Solve for $U(x,t)$

$$\begin{cases} U_t = k U_{xx} & \text{in } 0 < x < \infty \\ U(x,0) = e^{-x} \,, \; U(0,t) = 0 \end{cases}$$

Matlab: Plot the solution

5. Solve for $U(x,t)$

$$\begin{cases} U_{tt} = c^2 U_{xx} & \text{for } 0 < x < L \\ U(x,0) = 0 \,, \quad U_t(x,0) = x \\ U(0,t) = U(L,t) = 0 \end{cases}$$

Matlab:
Plot the solution for $L = \pi$ and $c = 5$.

6. Solve for $U(x,t)$

$$\begin{cases} U_{tt} = U_{xx} + \cos x \,, & \text{in } \mathbb{R} \times (0,\infty) \\ U(x,0) = \sin x \,, \; U_t(x,0) = 1 + x \end{cases}$$

Matlab: Plot the solution

7. A rod has length $L=1$ and constant $k=1$. Its temperature satisfies the heat equation. Its left end is held at temperature 0, its right end at temperature 1. Initially (at $t=0$) the temperature is given by

$$\phi(x) = \begin{cases} \frac{5x}{2} & 0 < x < \frac{2}{3} \\ 3-2x & \frac{2}{3} < x < 1 \end{cases}$$

Find the solution including the coefficients.

MatCalc: Plot the solution

8. Solve for $U(x,t)$

$$\begin{cases} U_t = U_{xx} & \text{in } (0,1) \\ U_x(0,t) = 0, \quad U_x(1,t) = 1 \\ U(x,0) = x^2 \end{cases}$$

Compute the coefficients explicitly.

Find the limit

$$\lim_{t \to \infty} U(x,t)$$

9. Solve for $u(x,t)$

$$u_{tt} = u_{xx} + e^t \sin 5x \quad \text{for} \quad 0 < x < \pi$$

$$u(0,t) = u(\pi,t) = 0$$

$$u(x,0) = 0, \; u_t(x,0) = \sin 3x$$

Matlab: Plot the solution.

10. Solve for $u(x,t)$

$$\begin{cases} u_t = u_{xx} \text{ in } (0,L) \\ u(0,t) = 0, \; u(L,t) = At \\ u(x,0) = 0 \end{cases}$$

where A is a constant

Matlab: Plot the solution for $L = \pi$ and $A = 1$