Midterm 2

MA 4335

Due 04/24/15

1. Solve

[20p]
$$\begin{cases} U_{tt} = 4U_{xx} & \text{in } 0 < x < \infty \\ U(x,0) = 0\,; \; U_t(x,0) = 1\,; \\ U_t(0,t) + 3U_x(0,t) = 0\,. \end{cases}$$

Discuss the discontinuity at origin! How does it propagate?

[5p] Plot the solution for several time values (e.g. $t=1$, $t=2$, $t=3$, $t=4$).

2. Solve

[20p]
$$U_{tt} = 9U_{xx} \qquad 0 < x < \infty$$
$$U(0,t) = t^2\,, \; U(x,0) = x\,, \; U_t(x,0) = 0$$

[5p] Plot your solution.

3. Solve.

20 ⌉
$$\begin{cases} U_{tt} = U_{xx} & \text{in } (0,1) \\ U(0,t) = 6 \;;\; U(1,t) = 3 \\ U(x,0) = \sin \pi x + 3\sin 7\pi x + 6 - 3x \\ U_t(x,0) = 0 \end{cases}$$

5 ⌉ Plot your solution.

4. Solve.

24 ⌉
$$\begin{cases} U_t = 2U_{xx} + x^2 - x - 4t & \text{in } (0,1) \\ U(0,t) = 0 = U(1,t) \\ U(x,0) = \sin \pi x + 6 \sin 8\pi x \end{cases}$$

5 ⌉ Plot your solution.

5. Solve

(20) $$\begin{cases} u_t = u_{xx} & \text{in } (0, \pi) \\ u_x(0,t) = 0 = u_x(\pi, t) \\ u(x,0) = \frac{-x^3}{3} + \frac{x^2}{2} \cdot \pi \end{cases}$$

(5) Plot your solution