Exam 1: Math 1431 Fall 2017
Professor William Ott

Exercise 1. (15; 5 each) Determine if each of the following statements is true or false.

(a) If f is continuous at a, then f is differentiable at a.

(b) The limit

$$\lim_{x \to 0} \frac{x^2}{|x|}$$

exists.

(c) If f and g are functions such that $f'(x) = g'(x)$ for all real x, then f and g must be the same function.

Exercise 2. (40; 10 each) Differentiate each of the following.

(a) $f(x) = x^2 e^x$

(b) $g(x) = \ln(\cos(7x))$

(c) $h(x) = \frac{\sqrt{x}}{1+\sin^2(x)}$

(d) $y = e^{x^2} + 5x^{4/3} - 11\pi$

Exercise 3. (10) Find the equation of the tangent line to the graph of $y = x + \frac{2}{x}$ at the point $(2, 3)$.

Exercise 4. (10) Find the horizontal and vertical asymptotes of the function f defined by

$$f(x) = \frac{7x + 15}{x^2 - 5x + 4}.$$

Exercise 5. (10) Use the intermediate value theorem to show that the equation $e^x = 3 - 2x$ has a solution in the interval $(0, 1)$.

Exercise 6. (10) Find the values of c and d for which the function

$$g(x) = \begin{cases} 4x, & \text{if } x < 2; \\ cx^2 + d, & \text{if } x \geqslant 2 \end{cases}$$

is differentiable at $x = 2$.

Exercise 7. (10) Suppose that two functions $f(x)$ and $g(x)$ are differentiable at a. Use the limit definition of the derivative to prove that the derivative of the product fg at a is given by

$$(fg)'(a) = f(a)g'(a) + f'(a)g(a).$$

Exercise 8. (Bonus 10) Define the function f by

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0; \\ 0, & \text{if } x = 0. \end{cases}$$

Is f differentiable at $x = 0$? If no, explain why not. If yes, find $f'(0)$ and justify your answer.