Exam 2: Math 1431 Fall 2017
Professor William Ott

Exercise 1. (15; 5 each) Determine if each of the following statements is true or false.

(a) If $f''(x) > 0$ for all x in (a, b), then f is increasing on (a, b).

(b) The function $f(x) = |x|$ satisfies the hypotheses of Rolle's theorem on the interval $[-5, 5]$.

(c) The function f defined by

$$f(x) = \begin{cases} \frac{\sin(x)}{x}, & \text{if } x \neq 0; \\ 1, & \text{if } x = 0 \end{cases}$$

is differentiable at $x = 0$.

Exercise 2. (20, 5 each) Let's study the function $f(x) = xe^{-x^2}$ defined on the real line.

(a) On what intervals is f increasing? On what intervals is f decreasing?

(b) List the critical point(s) of f. At which of these critical point(s) does f attain a local minimum?

(c) On what intervals is f concave up? Concave down?

(d) List the values of x at which f has an inflection point.

Exercise 3. (10) Find $\frac{dy}{dx}$ if $e^y \sin(x) = x + xy$.

Exercise 4. (10, 5 each) Evaluate the following limits.

(a) $\lim_{x \to \infty} \frac{(\ln(x))^2}{x}$

(b) $\lim_{x \to \infty} x^{1/x}$

Exercise 5. (10) A camera is located 50 feet from a straight road along which a car is traveling at 100 feet per second. The camera turns so that it is pointed at the car at all times. In radians per second, how fast is the camera turning as the car passes closest to the camera?

Exercise 6. (10) A cylindrical barrel (closed on top and bottom) is to hold a fixed volume V of liquid. Find the radius r that minimizes the surface area of the barrel.

Exercise 7. (10) Show that $2x + \cos(x) = 0$ has exactly one real solution.

Exercise 8. (10) Prove that

$$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1 + x^2}.$$