Assignment 1: Math 4320 Fall 2017
Professor William Ott

(1) (Exercise 1.2.3)

(a) Plot the distribution function

$$F(x) = \begin{cases} 0, & \text{for } x \leqslant 0; \\ x^3, & \text{for } 0 < x < 1; \\ 1, & \text{for } x \geqslant 1. \end{cases}$$

(b) Determine the corresponding density function $f(x)$ in the three regions (1) $x \leqslant 0$, (2) $0 < x < 1$, and (3) $1 \leqslant x$.

(c) What is the mean of the distribution?

(2) (Exercise 1.2.4) Let Z be a discrete random variable having possible values 0, 1, 2, and 3 and probability mass function

$$p(0) = \frac{1}{4}, \qquad p(1) = \frac{1}{2}, \qquad p(2) = \frac{1}{8}, \qquad p(3) = \frac{1}{8}.$$

(a) Plot the corresponding distribution function.

(b) Determine the mean $E[Z]$.

(c) Evaluate the variance $\mathrm{Var}[Z]$.

(3) (Exercise 1.2.7) Suppose X is a random variable having the probability density function

$$f(x) = \begin{cases} Rx^{R-1}, & \text{for } 0 \leqslant x \leqslant 1; \\ 0, & \text{elsewhere,} \end{cases}$$

where $R > 0$ is a fixed parameter.

(a) Determine the distribution function $F_X(x)$.

(b) Determine the mean $E[X]$.

(c) Determine the variance $\mathrm{Var}[X]$.

(4) (Problem 1.2.1) Thirteen cards numbered $1, 2, \ldots, 13$ are shuffled and dealt one at a time. Say a *match* occurs on deal k if the k^{th} card revealed is card number k. Let N be the total number of matches that occur in the thirteen cards. Determine $E[N]$.

Hint: Write $N = \mathbf{1}_{A_1} + \mathbf{1}_{A_2} + \cdots + \mathbf{1}_{A_{13}}$, where A_k is the event that a match occurs on deal k.

(5) (Problem 1.2.4) A fair coin is tossed until the first time that the same side appears twice in succession. Let N be the number of tosses required.

(a) Determine the probability mass function for N.

(b) Let A be the event that N is even and B be the event that $N \leqslant 6$. Evaluate $\Pr(A)$, $\Pr(B)$, and $\Pr(A \cap B)$.

(6) (Problem 1.2.6) A pair of dice is tossed. If the two outcomes are equal, the dice are tossed again, and the process repeated. If the dice are unequal, their sum is recorded. Determine the probability mass function for the sum.

(7) (Problem 1.2.8) Suppose X is a random variable with finite mean μ and variance σ^2, and $Y = a + bX$ for certain nonzero constants a and b. Determine the mean and variance for Y.

(8) (Problem 1.2.10) Random variables X and Y are independent and have probability mass functions

$$\begin{aligned} p_X(0) &= \frac{1}{2}, \quad & p_Y(1) &= \frac{1}{6}, \\ p_X(3) &= \frac{1}{2}, \quad & p_Y(2) &= \frac{1}{3}, \\ & & p_Y(3) &= \frac{1}{2}. \end{aligned}$$

Determine the probability mass function of the sum $Z = X + Y$.

(9) (Problem 1.2.13) Let X and Y be independent random variables each with the uniform probability density function

$$f(x) = \begin{cases} 1, & \text{for } 0 < x < 1; \\ 0, & \text{elsewhere.} \end{cases}$$

Find the joint probability density function of U and V, where $U = \max\{X, Y\}$ and $V = \min\{X, Y\}$.