Assignment 2: Math 4320 Fall 2017
Professor William Ott

(1) (Exercise 1.3.2) A fraction $p = 0.05$ of the items coming off a production process are defective. If a random sample of 10 items is taken from the output of the process, what is the probability that the sample contains exactly one defective item? What is the probability that the sample contains one or fewer defective items?

(2) (Exercise 1.3.4) A Poisson distributed random variable X has a mean of $\lambda = 2$. What is the probability that X equals 2? What is the probability that X is less than or equal to 2?

(3) (Problem 1.3.3) Let X be a Poisson random variable with parameter λ. Determine the probability that X is odd.

(4) (Problem 1.3.4) Let U be a Poisson random variable with mean μ. Determine the expected value of the random variable $V = 1/(1+U)$.

(5) (Problem 1.3.5) Let $Y = N - X$, where X has a binomial distribution with parameters N and p. Evaluate the product moment $E[XY]$ and the covariance $\mathrm{Cov}[X, Y]$.

(6) (Problem 1.3.9) Suppose X and Y are independent random variables with the geometric distribution

$$p(k) = (1-\pi)\pi^k$$

for $k = 0, 1, 2, \ldots$ (π in $(0,1)$ is fixed). Perform the appropriate convolution to identify the distribution of $Z = X + Y$ as a negative binomial.

(7) (Problem 1.3.13) Suppose that a sample of 10 is taken from a day's output of a machine that produces parts of which 5% are normally defective. If 100% of a day's production is inspected whenever the sample of 10 gives 2 or more defective parts, then what is the probability that 100% of a day's production will be inspected? What assumptions did you make?

(8) (Exercise 1.4.1) The lifetime, in years, of a certain class of light bulbs has an exponential distribution with parameter $\lambda = 2$. What is the probability that a bulb selected at random from this class will last more than 1.5 years? What is the probability that a bulb selected at random will last exactly 1.5 years?

(9) (Exercise 1.4.2) The median of a random variable X is any value a for which $\Pr(X \leqslant a) \geqslant \frac{1}{2}$ and $\Pr(X \geqslant a) \geqslant \frac{1}{2}$. Determine the median of an exponentially distributed random variable with parameter λ. Compare the median to the mean.

(10) (Exercise 1.4.6a) Suppose that U has a uniform distribution on the interval $[0, 1]$. Derive the density function for the random variable $Y = -\ln(1-U)$.

(11) (Exercise 1.4.7) Given independent exponentially distributed random variables S and T with common parameter λ, determine the probability density function of the sum $R = S + T$ and identify its type by name.

(12) (Problem 1.4.3) Let X and Y be independent random variables uniformly distributed over the interval $[\theta - 1/2, \theta + 1/2]$ for some fixed θ. Show that $W = X - Y$ has a distribution that is independent of θ with density function

$$f_W(w) = \begin{cases} 1 + w, & \text{for } -1 \leqslant w < 0; \\ 1 - w, & \text{for } 0 \leqslant w \leqslant 1; \\ 0, & \text{for } |w| > 1. \end{cases}$$

(13) (Problem 1.4.5) If X follows an exponential distribution with parameter $\alpha = 2$, and independently, Y follows an exponential distribution with parameter $\beta = 3$, what is the probability that $X < Y$?