Math 4320 Assignment 3

September 9, 2016

Exercise 1.5.2: A jar has four chips colored red, green, blue, and yellow. A person draws a chip, observes its color, and returns it. Chips are now drawn repeatedly, without replacement, until the first chip drawn is selected again. What is the mean number of draws required?

Exercise 1.5.3: Let X be an exponentially distributed random variable with parameter λ. Determine the mean of X
E.1.5.3.a: by integrating by parts in the definition in equation (**1.7**) with $m = 1$;
E.1.5.3.b: by integrating the upper tail probabilities in accordance with equation (**1.50**).
Which method do you find easier? (see last page for equations (**1.7**) and (**1.50**))

Problem 1.5.2: Let $X_1, X_2, ..., X_n$ be independent random variables, all exponentially distributed with the same parameter λ. Determine the distribution function for the minimum $Z = \min\{X_1, ..., X_n\}$.

Problem 1.5.3: Suppose that X is a discrete random variable having the geometric distribution whose probability mass function is

$$Pr(X = k) = p(1-p)^k \text{ for } k = 0, 1, ...$$

P.1.5.3.a: Determine the upper tail probabilities $Pr(X > k)$ for $k = 0, 1, ...$
P.1.5.3.b: Evaluate the mean via $E[X] = \sum_{k \geq 0} Pr(X > k)$.

Exercise 2.1.1: I roll a six-sided die and observe the number N on the uppermost face. I then toss a fair coin N times and observe X, the total number of heads to appear. What is the probability that $N = 3$ and $X = 2$? What is the probability that $X = 5$? What is $E[X]$, the expected number of heads to appear?

Exercise 2.1.2: Four nickels and six dimes are tossed, and the total number N of heads is observed. If $N = 4$, what is the conditional probability that exactly two of the nickels were heads?

Exercise 2.1.5: Let X be a Poisson random variable with parameter λ. Find the conditional mean of X given that X is odd.

Problem 2.1.4: Suppose that X has a binomial distribution with parameters $p = \frac{1}{2}$ and N, where N is also random and follows a binomial distribution with parameters $q = \frac{1}{4}$ and $M = 20$. What is the mean of X?

Problem 2.1.5: A nickel is tossed 20 times in succession. Every time that the nickel comes up heads, a dime is tossed. Let X count the number of heads appearing on tosses of the dime. Determine $Pr(X = 0)$.

Problem 2.1.6: A dime is tossed repeatedly until a head appears. Let N be the trial number on which this first head occurs. Then, a nickel is tossed N times. Let X count the number of times that the nickel comes up tails. Determine $Pr(X = 0), Pr(X = 1)$, and $E[X]$.

Problem 2.1.9: Let N have a Poisson distribution with parameter $\lambda = 1$. Conditioned on $N = n$, let X have uniform distribution over the integers $0, 1, ..., n + 1$. What is the marginal distribution for X?

Equation 1.7:

$$E[X^m] = \int_{-\infty}^{\infty} x^m f(x) dx.$$

Equation 1.50:

$$E[X] = \int_{0}^{\infty} [1 - F(z)] dz.$$