Math 4320 Fall 2017: Exam 1 Review
Professor William Ott

Exam 1 will cover the material in Sections 1.2–1.4, 1.5 (only tail probabilities), 2.1, 2.3.2, and 3.1–3.4 of *An Introduction to Stochastic Modeling* (Edition 4) by Pinsky and Karlin. Possible exercise types include true/false questions, statements of definitions and major results, computational exercises, and exercises requiring theoretical arguments. At least one of the exercises from Section 6 and at least one of the theoretical arguments in Section 5 will appear on Exam 1.

1. Definitions

You should be able to define and use the following.

(1) Sample space
(2) Events, independent events
(3) Random variables
  (a) Distribution function of a random variable
  (b) Probability mass function of a discrete random variable
  (c) Probability density function of a continuous random variable
  (d) Expected value
  (e) Variance, second moment
(4) Joint distribution function of a pair (X, Y) of random variables
(5) Independence of random variables
(6) Conditional probability of one event given a second event
(7) Conditional probability mass function
(8) Conditional expectation
(9) Random sum
(10) Discrete-time Markov chains, transition probability matrices

2. Distributions

(1) Discrete distributions: Bernoulli, binomial, geometric, Poisson
(2) Continuous distributions: uniform, exponential, normal

3. Computational techniques

(1) Law of total probability
(2) Use of tail probabilities to compute probabilities and expected values (see Section 1.5.1)
(3) Conditioning in the discrete case: conditional probability mass functions and the law of total probability (see Section 2.1). Conditioning is used to compute marginal distributions.
(4) Properties of $E[\cdot]$ and $\mathrm{Var}[\cdot]$
  (a) For any random variables X, Y and any scalars a, b, $E[aX + bY] = aE[X] + bE[Y]$.
  (b) For *independent* random variables X and Y, $\mathrm{Var}[X + Y] = \mathrm{Var}[X] + \mathrm{Var}[Y]$.
(5) Use transition probability matrices to compute probabilities of events for discrete-time Markov chains.
(6) If the row vector $\boldsymbol{x}$ is the initial probability distribution of a Markov chain, then $\boldsymbol{x}\mathsf{P}^n$ is the probability distribution after n steps. Here P is the transition matrix of the Markov chain.
(7) First-step analysis

4. Theoretical results

You should know and be able to apply the following.

(1) Theorem 3.1

5. Proofs/derivations

(1) Derive the formula for the expected value of a random sum (see pg. 60).

(2) Prove Theorem 3.1. The proof is the three-line computation on pg. 84.

6. Suggested problems

Please study the solutions to Assignments 1–4. Focus on the exercises that are not too long for an exam setting and on the problems that are not too difficult for an exam setting.