Math 4320 Fall 2017: Exam 2 Review
Professor William Ott

Exam 2 will cover the material in Sections 3.1–3.5, 3.8, 3.9.1, 4.1–4.2, 4.3.3, 5.1, 5.3, and 6.1 of *An Introduction to Stochastic Modeling* (Edition 4) by Pinsky and Karlin. Possible exercise types include true/false questions, statements of definitions and major results, computational exercises, and exercises requiring theoretical arguments. At least one of the exercises from Section 6 and at least one of the theoretical arguments in Section 5 will appear on Exam 2.

1. Definitions

You should be able to define and use the following.

(1) Discrete-time Markov chains, transition probability matrices
(2) Important types of Markov chains: successive maxima, one-dimensional random walks, success runs
(3) Branching processes
(4) Regular transition probability matrices
(5) Limiting distribution of a regular transition probability matrix and interpretations
(6) Recurrent state, transient state
(7) Poisson process, nonhomogeneous Poisson process
(8) Cox process

2. Distributions

(1) Discrete distributions: Bernoulli, binomial, geometric, Poisson
(2) Continuous distributions: uniform, exponential, normal

3. Techniques

(1) Law of total probability
(2) Conditioning
(3) Use transition probability matrices to compute probabilities of events for discrete-time Markov chains.
(4) If the row vector $\boldsymbol{x}$ is the initial probability distribution of a Markov chain, then $\boldsymbol{x}\mathsf{P}^n$ is the probability distribution after n steps. Here P is the transition matrix of the Markov chain.
(5) Find the transition matrix when given a description of a Markov model. See the Erhenfest urn model and the exercises and problems in Section 3.3.
(6) **First step analysis**. This technique is used to find probabilities of absorption in selected absorbing states, mean times to absorption, et cetera.
(7) Computation of extinction probabilities for branching processes
  (a) Recursive formula for u_n (see pg. 150)
  (b) Definition of u_∞
  (c) u_∞ is the smallest positive solution of $\phi(s) = s$, where $\phi(s)$ is the generating function of the offspring distribution.
(8) Compute limiting distributions of regular transition probability matrices (Theorem 4.1)
(9) Express long run probabilities in terms of the limiting distribution and the transition matrix (for example, Problems 4.1.6 and 4.1.13)
(10) Determine if a state i is recurrent or transient (using Theorem 4.2)

(11) Solve problems involving Poisson processes, nonhomogeneous Poisson processes, and Cox processes

4. Theoretical results

You should know and be able to apply the following.

(1) Theorem 3.1
(2) Theorem 4.1
(3) Theorem 4.2
(4) Theorems 5.1 and 5.2 (basic facts about the Poisson distribution)
(5) Theorems 5.4, 5.5, and 5.6 (distributions associated with the Poisson process)

5. Proofs/derivations

(1) Show that for the one-dimensional random walk, state 0 is recurrent if $p = q = 1/2$. (I will give you Stirling's formula.)
(2) Given a Poisson process with rate parameter $\lambda > 0$, show that the first two inter-arrival times are independent random variables, each exponentially distributed with mean $\frac{1}{\lambda}$.
(3) Theorem 5.6

6. Suggested problems

Please study the solutions to Assignments 4–7. Focus on the exercises that are not too long for an exam setting and on the problems that are not too difficult for an exam setting.