Math 4320 Fall 2017 – Exam 3 Solutions

(1) [a] False [b] True [c] True [d] False

(2) [a] $\frac{(7\lambda)^3}{3!} e^{-7\lambda}$ [b] $\frac{(5\lambda)^2}{2!} e^{-5\lambda}$ [c] $\left[\frac{(2\lambda)^1}{1!} e^{-2\lambda}\right]\left[\frac{(5\lambda)^2}{2!} e^{-5\lambda}\right]$

[d] $\int_a^b \lambda e^{-\lambda t}\,dt = -e^{-\lambda t}\Big|_a^b = -e^{-\lambda b} + e^{-\lambda a}$.

[e] $\frac{1}{5}$ [f] $\int_0^1 \int_{w_1}^1 \frac{2}{9}\,dw_2\,dw_1 = 2/9\left(\frac{1}{2}\right) = \boxed{\frac{1}{9}}$.

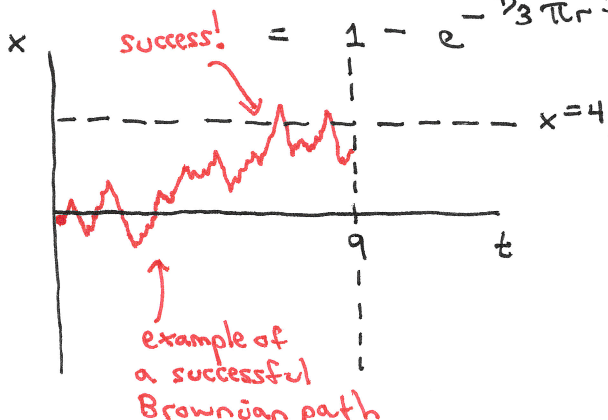
(3) $\Pr\{D \le r\} = 1 - \Pr\{D > r\}$

$= 1 - \Pr\{$0 particles in sphere of radius $r\}$

$= 1 - \frac{[\frac{4}{3}\pi r^3 \cdot \nu]^0}{0!} e^{-\frac{4}{3}\pi r^3 \nu}$

$= 1 - e^{-\frac{4}{3}\pi r^3 \nu}$.

(4)



Using the reflection principle,

$\Pr\{\max_{0\le t\le 9} B(t) \ge 4\}$

$= 2\cdot \Pr\{B(9) > 4\}$

$= 2\cdot \int_4^\infty \frac{1}{\sqrt{2\pi}\cdot 3} e^{-x^2/2\cdot 9}\,dx$.

(5) [a] $F(s) = \Pr\{W_1 \le s \mid X(T)=1\} = \dfrac{\left[\frac{(\lambda s)^1}{1!} e^{-\lambda s}\right]\left[\frac{[\lambda(T-s)]^0}{0!} e^{-\lambda(T-s)}\right]}{\frac{(\lambda T)^1}{1!} e^{-\lambda T}}$

$= s/T$.

[b] $\frac{d}{ds} F(s) = \frac{1}{T}$.

(6) $\Pr\{$system survives at time $t\}$

$= \sum_{k=0}^\infty \Pr\{$system survives at time $t \mid k$ shocks$\}\cdot \Pr\{k$ shocks$\}$

$= \sum_{k=0}^\infty \alpha^k \left[\frac{(\lambda t)^k}{k!} e^{-\lambda t}\right]$

optional additional simplification:

$= e^{-\lambda t} \sum_{k=0}^\infty \frac{(\alpha\lambda t)^k}{k!}$

$= e^{-\lambda t} e^{\alpha\lambda t}$

$= e^{\lambda t(\alpha - 1)}$ □