Math 3325 Fall 2019

Assignment 2 Solutions

$\boxed{1.5.3d}$ Let x and y be integers. We prove that if xy is even, then x or y is even. We proceed by contraposition. Assume x and y are both odd. Then $x = 2k+1$ and $y = 2l + 1$ for some integers k and l. We have

$$\begin{aligned} xy &= (2k+1)(2l+1) \\ &= 4kl + 2k + 2l + 1 \\ &= 2(2kl + k + l) + 1. \end{aligned}$$

Since $2kl + k + l$ is an integer, we conclude that xy is odd. $\square$

$\boxed{1.5.3g}$ This one will not be graded, but let's discuss it!

$\boxed{1.5.5c}$ A circle has center $(2,4)$. Prove that if $(0,3)$ is not inside the circle, then $(3,1)$ is not inside the circle.

Proof. Let r be the radius of the circle. Since $(0,3)$ is not inside the circle, we have

$$\begin{aligned} r &\le \text{distance}((2,4),(0,3)) \\ &= \sqrt{(2-0)^2 + (4-3)^2} \\ &= \sqrt{5}. \end{aligned}$$

But distance$((2,4),(3,1)) = \sqrt{10} > r$, so $(3,1)$ is not inside the circle. $\square$

$\boxed{1.5.9}$ If n is a natural number, then $n/(n+1) > n(\frac{1}{n+2})$. Proceeding by way of contradiction, assume

$$\frac{n}{n+1} \le \frac{n}{n+2}.$$

Then $n^2 + 2n \le n^2 + n$, so $n \le 0$. But $n > 0$, since n is a natural number. This is a contradiction. $\square$

$\boxed{1.5.11}$ Suppose x, y, z satisfy $0 < x < y < z < 1$. Prove that at least two of x, y, z are within $1/2$ unit from one another. By way of contradiction, assume that $y - x \ge \frac{1}{2}$ and $z - y \ge \frac{1}{2}$. Then

$$\begin{aligned} 1 &= x + (y-x) + (z-y) + (1-z) \\ &\ge x + \tfrac{1}{2} + \tfrac{1}{2} + (1-z), \end{aligned}$$

so $0 \ge x + (1-z)$. But $x + (1-z) > 0$, a contradiction.

1.5.7 a Let a, b, c be positive integers. Prove that $ac \mid bc$ if and only if $a \mid b$.

($\Rightarrow$) Assume $ac \mid bc$. Then $bc = k(ac)$ for some integer k. Since $c \neq 0$, we divide by c to obtain $b = ak$. Thus $a \mid b$.

($\Leftarrow$) Assume $a \mid b$. Then $b = am$ for some integer m. Multiplying by c gives $bc = a(mc) = (ac)m$. Consequently, $ac \mid bc$. $\square$