Math 3325 Fall 2019
Assignment 3 Solutions

1.6.1g For every odd integer m, if $m = 4k+1$ for some integer k, then $m+2$ has the form $4j-1$ for some integer j.

Proof: Let m be an odd integer and suppose that $m = 4k+1$ for some integer k. Then

$$\begin{aligned} m+2 &= (4k+1)+2 \\ &= 4k+3 \\ &= 4k+4-1 \\ &= 4(k+1)-1 \\ &= 4j-1, \end{aligned}$$

where $j = k+1$. We have shown that $m+2$ has the desired form. □

1.6.4g The statement is false. For instance, when $x=1$, $2^1 = 2$ and $1+1 = 2$, so $2^x > x+1$ fails for $x=1$. □

1.7.11c Common divisors: $-18, -9, -6, -3, -2, -1, 1, 2, 3, 6, 9, 18$

$\gcd(18,-54) = 18$

1.7.14c If $a \mid bc$ and $\gcd(a,b)=1$, then $a \mid c$.

Proof: Assume $a \mid bc$ and $\gcd(a,b)=1$. By the Bezout lemma, there exist integers x and y such that

$$ax + by = 1.$$

Multiplying by c gives $cax + cby = c$. Since $a \mid bc$, $bc = aq$ for some integer q. Substituting gives

$$cax + aqy = c, \text{ or}$$
$$a(cx+qy) = c.$$

Since $cx+qy$ is an integer, we conclude that $a \mid c$. □

1.7.18 Proof: Since a,b are relatively prime, by the Bezout lemma there exist integers x,y such that $ax+by=1$. Now let c be an integer. Multiplying by c gives $cax + cby = c$, or $a(cx) + b(cy) = c$. We have expressed c as an integer linear combination of a,b. □