Math 3325 Fall 2019
Assignment 4 Solutions

2.1.7 Prove that if $x \notin B$ and $A \subset B$, then $x \notin A$.
Proof: We proceed by contradiction. Suppose $x \notin B$, $A \subset B$, and $x \in A$. Since $x \in A$ and $A \subset B$, we have that $x \in B$. But now $x \in B$ and $x \notin B$! Contradiction.

2.1.13 Let $a, b \in \mathbb{N}$. Prove that $a = b$ iff $a\mathbb{Z} = b\mathbb{Z}$.
Proof: ($\Rightarrow$) If $a = b$, then $a\mathbb{Z} = b\mathbb{Z}$ by definition.
($\Leftarrow$) Suppose $a\mathbb{Z} = b\mathbb{Z}$. Since $a\mathbb{Z} \subset b\mathbb{Z}$ and $a \in a\mathbb{Z}$ since $a = a \cdot 1$, it follows that $a \in b\mathbb{Z}$, so $a = bk$ for some positive integer k. Similarly, $b \in b\mathbb{Z}$ and $b\mathbb{Z} \subset a\mathbb{Z}$, so $b \in a\mathbb{Z}$ and therefore $b = al$ for some positive integer l. Substituting, we have $a = bk = (al)k$, or $1 = lk$. This forces $k = 1$ and $l = 1$, yielding $a = b$. □

2.2.3 bc (b) $\mathbb{Z}^+ \cap D$ (c) D

2.2.9 b Prove that $A \subset B \cup C$ and $A \cap B = \emptyset$, then $A \subset C$.
Proof: Assume $A \subset B \cup C$ and $A \cap B = \emptyset$. Let $x \in A$. Since $A \subset B \cup C$, $x \in B \cup C$, so $x \in B$ or $x \in C$. But $x \in A$ and $A \cap B = \emptyset$, so $x \notin B$. We conclude that $x \in C$. □

2.2.10 a Prove that if $C \subset A$ and $D \subset B$, then $C \cap D \subset A \cap B$.
Proof: Assume $C \subset A$ and $D \subset B$. Let $x \in C \cap D$. Since $x \in C$ and $C \subset A$, we have $x \in A$. Since $x \in D$ and $D \subset B$, we have $x \in B$. Consequently, $x \in A \cap B$. The proof that $C \cap D \subset A \cap B$ is complete. □