Math 3325 Fall 2019

Assignment 7 Solutions

(3.2.5.i) Define T on $\mathbb{R}$ by xTy iff $\sin(x)=\sin(y)$. We claim that T is an equivalence relation.

Reflexive: Let $x \in \mathbb{R}$. Then xTx, since $\sin(x)=\sin(x)$.

Symmetric: Let $x, y \in \mathbb{R}$ and suppose $\sin(x)=\sin(y)$. Then $\sin(y)=\sin(x)$, so yTx.

Transitive: Let $x, y, z \in \mathbb{R}$ and suppose xTy and yTz. Then $\sin(x)=\sin(y)$ and $\sin(y)=\sin(z)$, so $\sin(x)=\sin(z)$ and therefore xTz.

- $[0] = \{ n\pi : n \in \mathbb{Z} \}$
- $[\pi/2] = \{ \pi/2 + 2\pi n : n \in \mathbb{Z} \}$
- $[\pi/4] = \{ \pi/4 + 2\pi n : n \in \mathbb{Z} \} \cup \{ 3\pi/4 + 2\pi n : n \in \mathbb{Z} \}$

(3.2.7.bd)

$\boxed{b}$ Reflexive, Symmetric, Transitive

$\boxed{d}$ Reflexive, Not symmetric, Not transitive

(3.2.11) Define S on $\mathbb{N}$ by xSy iff $3 \mid (x+y)$. Prove that S is not an equivalence relation.

Proof: We have $1S2$ and $2S1$, but 1 is not S-related to 1, since $3 \nmid 2$. Therefore S is not transitive. □

(3.2.15)

(a) We prove that $R \cup R^{-1}$ is symmetric. (Here R is a relation on the set A.) Suppose $(x,y) \in R \cup R^{-1}$.
If $(x,y) \in R$, then $(y,x) \in R^{-1}$, so $(y,x) \in R \cup R^{-1}$.
If $(x,y) \in R^{-1}$, then $(y,x) \in R$, so $(y,x) \in R \cup R^{-1}$.

(b) Suppose S is a symmetric relation on A and $R \subseteq S$. We show that $R^{-1} \subseteq S$. Let $(x,y) \in R^{-1}$. Then $(y,x) \in R$ and $R \subseteq S$, so $(y,x) \in S$. But S is symmetric, so $(x,y) \in S$. □

(3.3.7.bc)

(b) First, let $(x,y) \in \mathbb{R} \times \mathbb{R}$. Then $(x,y) \in A_a$, with $a = y + x^2$. Second, if $a_1 \neq a_2$, then $A_{a_1} \cap A_{a_2} = \emptyset$, since no point

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$(x, y) \in \mathbb{R} \times \mathbb{R}$ can simultaneously satisfy $y + x^2 = a_1$ and $y + x^2 = a_2$.

(c) The equivalence relation is given by

$$(x_1, y_1) \, R \, (x_2, y_2)$$

iff $y_1 + x_1^2 = y_2 + x_2^2$. $\square$