Exam 1: Math 3325 Fall 2019
Professor William Ott

Exercise 1. (10, 5 each) Determine if each of the following statements is true or false.

(a) Every nonempty subset of $\mathbb{N}$ has a least element.

(b) If a, b, and c are integers such that $a \neq 0$ and a divides bc, then a divides b or a divides c.

Exercise 2. (10) Let m be an integer. Prove that if m^2 is even, then m is even.

Exercise 3. (10, 5 each) Carefully define the following.

(a) The nonzero integers a and b are ***relatively prime***.

(b) The natural number d is the ***greatest common divisor*** of the nonzero integers a and b.

Exercise 4. (10) Let a, b, and c be positive integers. Prove that ac divides bc if and only if a divides b.

Exercise 5. (10) Prove by contradiction that if n is a natural number, then

$$\frac{n}{n+1} < \frac{n+1}{n+2}.$$

(This inequality is known as a 'baseball inequality'.)

Exercise 6. (10) Let a, b, and p be integers. Prove that if p is prime and p divides ab, then p divides a or p divides b. (This result is known as Euclid's lemma.)

Exercise 7. (15) For nonzero integers a and b, the integer n is called a ***common multiple*** of a and b if and only if a divides n and b divides n. We say the positive integer m is the ***least common multiple*** of a and b if and only if

(1) m is a common multiple of a and b, and
(2) if n is a positive common multiple of a and b, then $m \leqslant n$.

(a) **(5)** Find the least common multiple of 6 and 14.

(b) **(10)** Let a and b be natural numbers and let m be their least common multiple. Prove that $m \leqslant ab$.