Math 3325 Fall 2019: Exam 1 Review
Professor William Ott

Exam 1 will cover the material in Sections 1.4–1.7 of *A Transition to Advanced Mathematics (Seventh Edition)* by Smith, Eggen, and St. Andre. Possible exercise types include true/false questions, statements of definitions and major results, computational exercises, and exercises requiring theoretical arguments. At least one of the exercises from Section 5 and at least one of the theoretical arguments in Section 4 will appear on Exam 1.

1. Definitions

You should be able to define and use the following.

(1) Integer a divides integer b, written $a|b$.
(2) Integers: Odd, even, prime
(3) Divisor, quotient, and remainder (in the context of the division algorithm)
(4) Linear combinations
(5) Common divisor, greatest common divisor
(6) Relatively prime integers

2. Proof techniques

(1) General proof techniques
- **(a)** $P \Rightarrow Q$ by direct proof
- **(b)** $P \Rightarrow Q$ by contraposition
- **(c)** $(P \wedge Q) \Rightarrow R$
- **(d)** $(P \vee Q) \Rightarrow R$
- **(e)** $P \Rightarrow (Q \wedge R)$
- **(f)** $P \Rightarrow (Q \vee R)$
- **(g)** Proof by exhaustion
- **(h)** Proof by contradiction
- **(i)** $P \Leftrightarrow Q$: Prove $P \Rightarrow Q$ and $Q \Rightarrow P$ separately.
- **(j)** $P \Leftrightarrow Q$: Prove in a single step using a sequence of if and only if statements.
- **(k)** $(\forall x)P(x)$
  - By direct proof
  - By contradiction
- **(l)** Existence proofs
  - Constructive arguments
  - Non-constructive arguments (such as the intermediate value theorem and the mean value theorem)
  - By contradiction
- **(m)** Unique existence

(2) Well-ordering principle

3. Theoretical results

You should know and be able to apply the following.

(1) Division algorithm
(2) Theorem 1.7.1
(3) Bézout's lemma (Theorem 1.7.3)
(4) Theorem 1.7.4
(5) Euclid's lemma (Lemma 1.7.5)

4. Proofs/Derivations

(1) Prove that $\sqrt{2}$ is an irrational number.

(2) Prove that the set of prime numbers is an infinite set.

(3) Prove Euclid's lemma.

5. Suggested problems

Please study Assignments 1–3 to help you prepare for Exam 1. At least one of the graded assignment problems from Assignments 1–3 will appear on the exam.