Math 3325 Fall 2019

Exam 1 Solutions

E1 (a) True

(b) False

E2 Let m be an integer and assume m is odd. Then $m = 2k+1$ for some integer k, so $m^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$. We conclude that m^2 is odd. The proof by contraposition is complete.

E3 (a) The nonzero integers a and b are relatively prime if $\gcd(a, b) = 1$.

(b) $d \in \mathbb{N}$ is the greatest common divisor of the nonzero integers a and b if

(1) $d \mid a$ and $d \mid b$, and

(2) if $c \in \mathbb{Z}$ divides both a and b, then $c \leq d$.

E4 ($\Rightarrow$) Suppose $ac \mid bc$. Then $bc = ack$ for some integer k. Dividing by c gives $b = ak$, so $a \mid b$.

($\Leftarrow$) Suppose $a \mid b$. Then $b = al$ for some integer l. Multiplying by c gives $bc = acl$, so $ac \mid bc$.

E5 Suppose there exists a natural number n such that $\dfrac{n}{n+1} \geq \dfrac{n+1}{n+2}$. Cross-multiplying gives

$$n^2 + 2n \geq n^2 + 2n + 1, \text{ or } 0 \geq 1. \text{ Contradiction.}$$

E6 Suppose p is prime and $p \mid ab$, but p does not divide a. We must show that $p \mid b$. Since p is prime and p does not divide a, $\gcd(p, a) = 1$, so by Bezout there exist integers k, l such that

$$pk + al = 1.$$

Multiplying by b gives $pkb + abl = b$. Since $p \mid ab$, $ab = pm$ for some integer m, giving

$b = pkb + (pm)l =$ [redacted] $p(kb + ml)$.

We conclude that $p \mid b$.

E7 (a) 42 (b) Since $a \mid (ab)$ and $b \mid (ab)$, ab is a common multiple of a and b. Since $ab > 0$, $m \leq ab$. $\square$