Math 3325 Fall 2019: Exam 2 Review
Professor William Ott

Exam 2 will cover the material in Sections 2.1–2.5 and 3.1–3.3 of *A Transition to Advanced Mathematics (Seventh Edition)* by Smith, Eggen, and St. Andre. Possible exercise types include true/false questions, statements of definitions and major results, computational exercises, and exercises requiring theoretical arguments. At least one of the exercises from Section 6 and at least one of the theoretical arguments in Section 5 will appear on Exam 2.

1. Definitions

You should be able to define and use the following.

(1) Integer a divides integer b, written $a|b$.
(2) Integers: Odd, even, prime
(3) Divisor, quotient, and remainder (in the context of the division algorithm)
(4) Linear combinations
(5) Common divisor, greatest common divisor
(6) Relatively prime integers
(7) Set theory
  - **(a)** Empty set
  - **(b)** Subset ($A \subset B$)
  - **(c)** Equality of sets ($A = B$)
  - **(d)** Power set
  - **(e)** Operations: Union, intersection, difference, complement
  - **(f)** Disjoint sets
  - **(g)** Indexed families of sets

(8) Factorials
(9) Sigma notation, product notation
(10) Relations
  - **(a)** Cartesian product of sets
  - **(b)** Relation from A to B, relation on a set A
  - **(c)** Directed graph
  - **(d)** Reflexive, symmetric, transitive
  - **(e)** Equivalence relations
  - **(f)** Equivalence classes
  - **(g)** The equivalence relation $\equiv_m$ on $\mathbb{Z}$

(11) Partitions of sets

2. Proof techniques

(1) General proof techniques
  - **(a)** $P \Rightarrow Q$ by direct proof
  - **(b)** $P \Rightarrow Q$ by contraposition
  - **(c)** $(P \wedge Q) \Rightarrow R$
  - **(d)** $(P \vee Q) \Rightarrow R$
  - **(e)** $P \Rightarrow (Q \wedge R)$
  - **(f)** $P \Rightarrow (Q \vee R)$
  - **(g)** Proof by exhaustion
  - **(h)** Proof by contradiction
  - **(i)** $P \Leftrightarrow Q$: Prove $P \Rightarrow Q$ and $Q \Rightarrow P$ separately.
  - **(j)** $P \Leftrightarrow Q$: Prove in a single step using a sequence of if and only if statements.
  - **(k)** $(\forall x)P(x)$

   - By direct proof
   - By contradiction
- **(l)** Existence proofs
   - Constructive arguments
   - Non-constructive arguments (such as the intermediate value theorem and the mean value theorem)
   - By contradiction
- **(m)** Unique existence

(2) Set-theoretic proof techniques
- **(a)** Prove that $A \subset B$.
- **(b)** Let A and B be sets. Prove that $A = B$.

(3) Mathematical induction
- **(a)** Principle of mathematical induction (PMI)
- **(b)** Generalized principle of mathematical induction
- **(c)** Principle of complete induction (PCI)

(4) Well-ordering principle

(5) Determine, with proof, if a given relation is reflexive, symmetric, transitive.

3. Theoretical results

You should know and be able to apply the following.

(1) Division algorithm

(2) Theorem 2.1.5

(3) De Morgan laws (Theorem 2.2.2)

(4) Fundamental theorem of arithmetic (Theorem 2.5.3)

(5) $\equiv_m$ is an equivalence relation on $\mathbb{Z}$ (Theorem 3.2.2)

(6) Theorem 3.3.1

4. Computational techniques

(1) Compute unions and intersections of indexed families of sets.

(2) Draw the directed graph associated with a relation.

(3) Find equivalence classes.

5. Proofs/Derivations

(1) Prove that $\sqrt{2}$ is an irrational number.

(2) Prove that the set of prime numbers is an infinite set.

(3) Prove that every natural number greater than one has a prime factor. See pg. 115.

(4) Prove that $\equiv_m$ is an equivalence relation on $\mathbb{Z}$. This is Theorem 3.2.2 in the textbook.

6. Suggested problems

Please study Assignments 4–7 to help you prepare for Exam 2. At least one of the graded assignment problems from Assignments 4–7 will appear on the exam.