Math 3325 Fall 2019: Exam 3 Review
Professor William Ott

Exam 3 will cover the material in Sections 3.2, 3.3, 4.3–4.5, 5.1, 5.2, and 5.4 of *A Transition to Advanced Mathematics (Seventh Edition)* by Smith, Eggen, and St. Andre. Possible exercise types include true/false questions, statements of definitions and major results, computational exercises, and exercises requiring theoretical arguments. At least one of the exercises from Section 6 and at least one of the theoretical arguments in Section 5 will appear on Exam 3.

1. Definitions

You should be able to define and use the following.

(1) Set theory
- **(a)** Empty set
- **(b)** Subset ($A \subset B$)
- **(c)** Equality of sets ($A = B$)
- **(d)** Power set
- **(e)** Operations: Union, intersection, difference, complement
- **(f)** Disjoint sets
- **(g)** Indexed families of sets

(2) Relations
- **(a)** Cartesian product of sets
- **(b)** Relation from A to B, relation on a set A
- **(c)** Directed graph
- **(d)** Reflexive, symmetric, transitive
- **(e)** Equivalence relations
- **(f)** Equivalence classes
- **(g)** The equivalence relation $\equiv_m$ on $\mathbb{Z}$

(3) Partitions of sets

(4) Functions
- **(a)** A function $f : A \to B$ is a rule that assigns an element $f(a) \in B$ to every element $a \in A$. The set A is called the domain of the function. The set B is called the codomain, or target space, of the function.
- **(b)** Range of a function
- **(c)** Two functions f and g are equal if they have the same domain and if $f(x) = g(x)$ for every x in this domain.
- **(d)** Composition of functions
- **(e)** Inverse of a function
- **(f)** Onto (surjective) functions
- **(g)** One-to-one (injective) functions
- **(h)** One-to-one correspondences (bijective functions)
- **(i)** Images of sets, inverse images of sets (see pg. 220)

(5) Cardinality theory
- **(a)** Equivalence of sets (see pg. 234)
- **(b)** Finite sets, infinite sets, countably infinite sets, uncountable sets
- **(c)** Cardinality of a set A, written card(A)
- **(d)** Cardinal numbers $\aleph_0$ and $\boldsymbol{c}$
- **(e)** The expressions $\text{card}(A) = \text{card}(B)$, $\text{card}(A) \leqslant \text{card}(B)$, and $\text{card}(A) < \text{card}(B)$. See pg. 260.

2. Proof techniques

(1) General proof techniques
- **(a)** $P \Rightarrow Q$ by direct proof

- **(b)** $P \Rightarrow Q$ by contraposition
- **(c)** $(P \wedge Q) \Rightarrow R$
- **(d)** $(P \vee Q) \Rightarrow R$
- **(e)** $P \Rightarrow (Q \wedge R)$
- **(f)** $P \Rightarrow (Q \vee R)$
- **(g)** Proof by exhaustion
- **(h)** Proof by contradiction
- **(i)** $P \Leftrightarrow Q$: Prove $P \Rightarrow Q$ and $Q \Rightarrow P$ separately.
- **(j)** $P \Leftrightarrow Q$: Prove in a single step using a sequence of if and only if statements.
- **(k)** $(\forall x)P(x)$
  - By direct proof
  - By contradiction
- **(l)** Existence proofs
  - Constructive arguments
  - Non-constructive arguments (such as the intermediate value theorem and the mean value theorem)
  - By contradiction
- **(m)** Unique existence

(2) Set-theoretic proof techniques
- **(a)** Prove that $A \subset B$.
- **(b)** Let A and B be sets. Prove that $A = B$.

(3) Determine, with proof, if a given relation is reflexive, symmetric, transitive.

(4) Prove that a function is onto, one-to-one, or both.

(5) Prove that a given set has cardinality $\aleph_0$ or $\boldsymbol{c}$.

(6) Use the Cantor-Schröder-Bernstein theorem to prove that two given sets have the same cardinality.

3. Theoretical results

You should know and be able to apply the following.

(1) Theorem 2.1.5

(2) De Morgan laws (Theorem 2.2.2)

(3) $\equiv_m$ is an equivalence relation on $\mathbb{Z}$ (Theorem 3.2.2)

(4) Theorem 3.3.1

(5) Theorems 4.3.1, 4.3.2, 4.3.3, and 4.3.4

(6) Theorem 4.4.1

(7) A function $f : A \to B$ is invertible if and only if f is one-to-one and onto.

(8) Theorem 4.5.1

(9) Theorem 5.2.2

(10) Theorem 5.2.4

(11) Theorem 5.2.5

(12) Cantor-Schröder-Bernstein theorem (Theorem 5.4.3)

4. Computational techniques

(1) Draw the directed graph associated with a relation.

(2) Find equivalence classes.

(3) Compute images of sets and inverse images of sets.

5. Proofs/Derivations

(1) Prove that $\equiv_m$ is an equivalence relation on $\mathbb{Z}$. This is Theorem 3.2.2 in the textbook.

(2) Theorem 4.3.1
(3) Theorem 5.2.4

6. Suggested problems

Please study Assignments 7–9 to help you prepare for Exam 3. At least one of the graded assignment problems from Assignments 7–9 will appear on the exam.