Math 3325 Fall 2019 (1)

Exam 3 Solutions

E1 (a) $f : A \to B$ is one-to-one if for all $x_1, x_2 \in A$, $f(x_1) = f(x_2)$ implies $x_1 = x_2$.

(b) Two sets A and B are equivalent, written $A \approx B$, if $\exists$ a bijection $h : A \to B$.

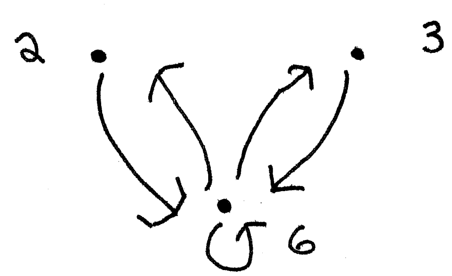
E2 (a) True

(b) False

(c) True

(d) True

(e) False



E3 (a) Look right.

(b) S is NOT reflexive. For instance, $(2,2) \notin S$.

(c) S IS symmetric. If xSy, then $x+y > 7$, so $y+x > 7$ and therefore ySx.

(d) S is NOT transitive. For instance, $2S6$ and $6S2$, but $(2,2) \notin S$.

E4 (a) $\aleph_0$ (b) $\aleph_0$ (c) c (d) c (e) c

E5 (a) The function a is one-to-one. Let $m, n \in \mathbb{N}$ and assume $2^m = 2^n$. Then $m \ln(2) = n \ln(2)$, and therefore $m = n$.

(b) The function a is not onto. 2^n is even for every $n \in \mathbb{N}$, so $3 \notin \text{Range}(a)$, for instance.

E6 Let $w \in C$. Since g is onto, $\exists$ $b \in B$ such that $g(b) = w$. Since f is onto, $\exists$ $a \in A$ such that $f(a) = b$. Then $(g \circ f)(a) = g(f(a)) = g(b) = w$, so $w \in \text{Range}(g \circ f)$.

E7 Define $f : \mathbb{N} \to E^+$ by $f(k) = 2k$. We show that f is a bijection. First, let $k_1, k_2 \in \mathbb{N}$ and suppose $f(k_1) = f(k_2)$. Then $2k_1 = 2k_2$, so $k_1 = k_2$. So f is one-to-one. Next, if $y \in E^+$, then $y/2 \in \mathbb{N}$ and $f(y/2) = y$. So f is onto.