Math 3325 Fall 2019

Functions

def A function $f : A \to B$ is a rule that assigns an output $f(a) \in B$ to every input $a \in A$. The set A is called the domain of f. The set B is called the codomain, or target space.

def Let $f : A \to B$ be a function. The range of f is defined as

$$\text{Range}(f) = \{ y \in B : f(x) = y \text{ for some } x \in A \}.$$

Note that $\text{Range}(f) \subset B$. These two sets may or may not be equal.

example Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$. Then $\text{Range}(f) = [0, \infty)$.

Proof. Let $y \in \mathbb{R}$. Then if $y \in [0, \infty)$, we have $\sqrt{y} \in \mathbb{R}$ and $f(\sqrt{y}) = (\sqrt{y})^2 = y$. We have shown that $[0, \infty) \subset \text{Range}(f)$. But $f(x) \geq 0$ for every $x \in \mathbb{R}$, so $\text{Range}(f) \subset [0, \infty)$. We conclude that $\text{Range}(f) = [0, \infty)$.

def Two functions f and g are equal if $\text{Domain}(f) = \text{Domain}(g)$ and if $f(x) = g(x)$ for every $x \in \text{Domain}(f)$.

some common functions

1. Let A be a nonempty set. The identity map $Id : A \to A$ is defined by $Id(x) = x$ for every $x \in A$.
2. Let $A \subset U$. The map $\chi_A : U \to \{0, 1\}$ is defined by $\chi_A(x) = \begin{cases} 1, & \text{if } x \text{ [redacted] } \in A; \\ 0, & \text{if } x \in U - A. \end{cases}$

   We call χ_A the characteristic function of A.
3. Let $f : A \to B$ be a function. Let $D \subset A$. The function $f|_D : D \to B$ defined by $f|_D(x) = f(x)$ for all $x \in D$ is called the restriction of f to D.