Exam 1: Math 1432 Spring 2018
Professor William Ott

Exercise 1. (40, 10 each) Evaluate the following indefinite integrals.

(a) $\int \sin^4(\theta)\cos^5(\theta)\,\mathrm{d}\theta$

(b) $\int \frac{3x+14}{x^2+2x}\,\mathrm{d}x$

(c) $\int \frac{1}{(1-x^2)^{3/2}}\,\mathrm{d}x$

(d) $\int \frac{1}{1+\sqrt{x}}\,\mathrm{d}x$

Exercise 2. (20, 10 each) Determine if the integral converges or diverges. If it converges, compute it.

(a) $\int_0^5 \frac{1}{(5-x)^{4/3}}\,\mathrm{d}x$

(b) $\int_1^\infty \frac{\ln(x)}{x^2}\,\mathrm{d}x$

Exercise 3. (10) Use the comparison test to determine if the following integral converges or diverges.

$$\int_{17}^{\infty} \frac{5+\sin(13x^2)}{x-e^{-x}}\,\mathrm{d}x$$

Exercise 4. (10) Derive the integration by parts formula

$$\int_a^b f(x)g'(x)\,\mathrm{d}x = f(x)g(x)\Big|_a^b - \int_a^b g(x)f'(x)\,\mathrm{d}x$$

using the product rule and the fundamental theorem of calculus.

Exercise 5. (10) Find the length of the curve defined by

$$y = f(x) = \int_1^x \sqrt{t^3-1}\,\mathrm{d}t$$

for $1 \leqslant x \leqslant 4$.

Exercise 6. (Bonus 10) Suppose $f(t)$ is continuous for $t \geqslant 0$. The Laplace transform of f is the function L defined by

$$L(s) = \int_0^\infty f(t)e^{-st}\,\mathrm{d}t.$$

Show that if $0 \leqslant f(t) \leqslant Me^{at}$ for all $t \geqslant 0$, where M and a are constants, then the Laplace transform $L(s)$ exists for all $s > a$.