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Math 2318
E1 Sp25
Sanders

1 Compute the following

a) $D+E = \begin{pmatrix} 2 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 3 \end{pmatrix}$

$= \boxed{\begin{pmatrix} 3 & 4 \end{pmatrix}}$

b) $A+2B = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix} + 2\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$

$= \boxed{\begin{pmatrix} 3 & 4 & 3 \\ 4 & 1 & 4 \end{pmatrix}}$

c) $AC = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

$= \boxed{\begin{pmatrix} 3 & 2 & 3 \\ 3 & 4 & 3 \end{pmatrix}}$

d) $DB = \begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$

$= \boxed{\begin{pmatrix} 3 & 2 & 3 \end{pmatrix}}$

②

2 Write as Augmented matrix and Solve from Echlon form.

a) $\left[\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 3 \\ 1 & 1 & 1 & 1 & 4 \end{array}\right]$ $R2 \to R2 - R1$

$\sim \left[\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 3 \\ 0 & -1 & 0 & 2 & 1 \end{array}\right]$ $R2 \to -R2$

$\left[\begin{array}{cccc|c} 1 & 2 & 1 & -1 & 3 \\ 0 & 1 & 0 & -2 & -1 \end{array}\right]$

free free

$x_3 = \alpha$ $x_4 = \beta$

$x_2 + 0\alpha - 2\beta = -1 \Rightarrow x_2 = -1 + 2\beta$

$x_1 + 2(-1+2\beta) + \alpha - \beta = 3$

$x_1 = 5 - \alpha - 3\beta$

$$\boxed{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \\ 0 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} -3 \\ 2 \\ 0 \\ 1 \end{pmatrix}}$$

(3)

2b
$$\left[\begin{array}{ccc|c} 3 & 1 & 1 & 4 \\ 2 & 0 & 1 & 2 \\ 1 & 1 & 1 & 2 \\ 3 & 1 & 2 & 4 \end{array}\right] \quad R1 \leftrightarrow R3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 0 & 1 & 2 \\ 3 & 1 & 1 & 4 \\ 3 & 1 & 2 & 4 \end{array}\right] \quad \begin{array}{l} R2 \to R2 - 2R1 \\ R3 \to R3 - 3R1 \\ R4 \to R4 - 3R1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -2 & -1 & -2 \\ 0 & -2 & -2 & -2 \\ 0 & -2 & -1 & -2 \end{array}\right] \quad \text{mult by } -1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 2 & 1 & 2 \\ 0 & 2 & 2 & 2 \\ 0 & 2 & 1 & 2 \end{array}\right] \quad \begin{array}{l} R3 \to R3 - R2 \\ R4 \to R4 - R2 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$0 = 0$ soln ok

$x_3 = 0$

$2x_2 + 0 = 2 \quad x_2 = 1$

$x_1 + 1 + 0 = 2 \quad x_1 = 1$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

3a

is $\begin{pmatrix}3\\1\\2\end{pmatrix} \overset{?}{\in} \text{Span}\left\{\begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}2\\1\\1\end{pmatrix}, \begin{pmatrix}3\\2\\1\end{pmatrix}\right\}$

i.e, is this solvable

$$\begin{pmatrix}3\\1\\2\end{pmatrix} = \alpha\begin{pmatrix}1\\1\\0\end{pmatrix} + \beta\begin{pmatrix}2\\1\\1\end{pmatrix} + \gamma\begin{pmatrix}3\\2\\1\end{pmatrix}$$

equiv $\begin{pmatrix}1 & 2 & 3\\1 & 1 & 2\\0 & 1 & 1\end{pmatrix}\begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix} = \begin{pmatrix}3\\1\\2\end{pmatrix}$

Use elimination

$$\left[\begin{array}{ccc|c}1 & 2 & 3 & 3\\1 & 1 & 2 & 1\\0 & 1 & 1 & 2\end{array}\right] \sim \left[\begin{array}{ccc|c}1 & 2 & 3 & 3\\0 & +1 & +1 & +2\\0 & 1 & 1 & 2\end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c}1 & 2 & 3 & 3\\0 & 1 & 1 & 2\\0 & 0 & 0 & 0\end{array}\right] \Leftarrow \text{Solvable}$$

$\alpha_3 = \alpha$

$\alpha_2 + \alpha = 2 \quad \alpha_2 = 2 - \alpha$

$\alpha_1 + 2(2-\alpha) + 3\alpha = 3 \quad \alpha_1 = -1 - \alpha$

(over)

for any $\alpha \in \mathbb{R}$

$$\boxed{\begin{pmatrix}3\\1\\2\end{pmatrix} = (1-\alpha)\begin{pmatrix}1\\1\\0\end{pmatrix} + (2-\alpha)\begin{pmatrix}2\\1\\1\end{pmatrix} + \alpha\begin{pmatrix}3\\2\\1\end{pmatrix}}$$

b) $\begin{pmatrix}2\\1\\2\end{pmatrix} \overset{?}{\in} \operatorname{Span}\left\{\begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}2\\1\\1\end{pmatrix}, \begin{pmatrix}3\\2\\1\end{pmatrix}\right\}$

Same as before

$$\left[\begin{array}{ccc|c}1 & 2 & 3 & 2\\ 1 & 1 & 2 & 1\\ 0 & 1 & 1 & 2\end{array}\right] \sim \left[\begin{array}{ccc|c}1 & 2 & 3 & 2\\ 0 & 1 & 1 & 1\\ 0 & 1 & 1 & 2\end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c}1 & 2 & 3 & 2\\ 0 & 1 & 1 & 1\\ 0 & 0 & 0 & 1\end{array}\right] \qquad \begin{pmatrix}2\\1\\2\end{pmatrix} \notin V.$$

NO Solution

(6)

4 Cast in Span of Std basis.

a) ~~Spanning~~ write set as $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

Std basis set

$\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$

so $\boxed{\dim(V_a) = 2.}$

b) write spanning vectors as $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 3 & 2 & 3 \end{bmatrix}$

reduce to canonical form

$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (over)

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Std basis ~~set~~ is $\boxed{\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}}$

$\boxed{\dim(V_b) = 2}$

5 $\mathcal{L}$ defined as given.

$N(\mathcal{L}) = \{ x \in dom(\mathcal{L}) : \mathcal{L}(\vec{x}) = \vec{0} \}$

$R(\mathcal{L}) = \{ y = \mathcal{L}(x) \mid x \in Dom \}$

a $\mathcal{L}(x) \equiv \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 1 & 3 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

Solve $A\vec{x} = \vec{0}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 2 & 5 & 7 & 0 \\ 1 & 3 & 4 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \alpha \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

(over)

$x_3 = \alpha \quad x_2 = -\alpha, \quad x_1 = -\alpha$

$$N(\mathcal{L}) = \text{span}\left\{\begin{pmatrix}-1\\-1\\1\end{pmatrix}\right\}$$

$$\text{Range}(\mathcal{L}) = \text{span}\left\{\begin{pmatrix}1\\2\\1\end{pmatrix}, \begin{pmatrix}2\\5\\3\end{pmatrix}, \begin{pmatrix}3\\7\\4\end{pmatrix}\right\}$$

Range of A is span of A's columns

Std basis

$$\begin{bmatrix}1 & 2 & 1\\2 & 5 & 3\\3 & 7 & 4\end{bmatrix} \sim \begin{bmatrix}1 & 2 & 1\\0 & 1 & 1\\0 & 1 & 1\end{bmatrix}$$

$$\sim \begin{bmatrix}1 & 2 & 1\\0 & 1 & 1\\0 & 0 & 0\end{bmatrix} \sim \begin{bmatrix}1 & 0 & -1\\0 & 1 & 1\\0 & 0 & 0\end{bmatrix}$$

Std basis for range is $\boxed{\left\{\begin{pmatrix}1\\0\\-1\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}\right\}}$

and $\text{Range}(\mathcal{L}) = \text{span}\left\{\begin{pmatrix}1\\0\\-1\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}\right\}$

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h) $\mathcal{L}(x) \equiv \begin{pmatrix} 1 & 1 & 1 & 2 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$

$N(\mathcal{L})$ solve $A\vec{x} = \vec{0}$

$$\begin{array}{cccc} 2 & 2 & 2 & 4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 3 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 2 & 1 & 0 \end{array}\right]$$

free

$x_4 = \alpha$

$2x_3 + \alpha = 0$

$x_3 = -\alpha/2$

$x_2 + 2\alpha = 0$

$x_2 = -2\alpha$

$x_1 + (-2\alpha) + (-\alpha/2) + 2\alpha = 0$

$x_1 = \frac{1}{2}\alpha$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \alpha \begin{pmatrix} 1/2 \\ -2 \\ -1/2 \\ 1 \end{pmatrix}$$

$N(\mathcal{L})$ has basis $\boxed{\left\{ \begin{pmatrix} 1/2 \\ -2 \\ -1/2 \\ 1 \end{pmatrix} \right\}}$

$\text{Range}(\mathcal{L}) = \text{span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \right\}$

(over)

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Std basis:

$$\begin{array}{c} \begin{matrix} 2 & 2 & 4 \end{matrix} \\ \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 2 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{Ran}_S(\mathcal{L}) = \mathrm{span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

The std basis set.