Math 2318 Exam 1. Sanders Spring 2025

This exam has 5 problems, and all 5 problems will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, **last name first**, and **student id number** on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless otherwise indicated.

1. Consider the matrices.

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad \begin{matrix} D = \begin{pmatrix} 2 & 1 \end{pmatrix} \\ E = \begin{pmatrix} 1 & 3 \end{pmatrix} \end{matrix}$$

Compute the following when defined.

(a) $D + E$ (b) $A + 2B$ (c) AC (d) DB

2. Write each of the following linear systems as an augmented matrix. Reduce to echelon form by Gaussian elimination. Finally determine <u>all</u> solutions if there is one.

(a) $$\begin{aligned} x_1 + 2x_2 + x_3 - x_4 &= 3 \\ x_1 + x_2 + x_3 + x_4 &= 4 \end{aligned}$$

(b) $$\begin{aligned} 3x_1 + x_2 + x_3 &= 4 \\ 2x_1 + x_3 &= 2 \\ x_1 + x_2 + x_3 &= 2 \\ 3x_1 + x_2 + 2x_3 &= 4 \end{aligned}$$

3. Answer the following.

Let $\mathcal{V} = \operatorname{span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \right\}$. (a) Is $\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \in \mathcal{V}$? (b) Is $\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \in \mathcal{V}$?

If true, write the given vector as a linear combination of the two spanning vectors. Please show all your work.

4. Consider the following two subspaces of $\mathbb{R}^3$.

(a) $\mathcal{V}_a = \operatorname{span}\left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \right\}$ (b) $\mathcal{V}_b = \operatorname{span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix} \right\}$

For each subspace, determine its **standard basis**, and also state its **dimension**.

5. Consider the following two matrices A each which defines a linear operator via matrix multiplication; i.e. $\mathcal{L}(\mathbf{x}) = A\mathbf{x}$. In part (a) $\mathcal{L} : \mathbb{R}^3 \to \mathbb{R}^3$ and in (b) $\mathcal{L} : \mathbb{R}^4 \to \mathbb{R}^3$.

(a) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 1 & 3 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 1 & 1 & 2 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{pmatrix}$

Determine a basis for $\mathcal{L}$'s **null space** and also compute the <u>standard basis</u> for $\mathcal{L}$'s **range space**.