Math 3331 Exam 1. Sanders Spring 2025

This exam has five problems, and all five will be graded. Use my supplied paper only. Return your solution sheets with the problems in order. Put your name, **last name first**, and **student id number** on each solution sheet you turn in. Each problem is worth 20 points with parts equally weighted unless indicated otherwise.

1. We've considered four basic classes of first order ODE's we can in principle solve. They are: (1) Separable, (2) Linear, (3) Homogeneous, (4) Exact. Classify each of the following but do not solve. An equation may be in more than one class (and you must list all classes if so), or it may be in none. No guessing – show your work!

(a) $x^2\dfrac{du}{dx} + u^2 + x^2 = 0$ (c) $u\dfrac{du}{dx} + x = 0$

(b) $x^2\dfrac{du}{dx} + 2xu + x^2 = 0$ (d) $e^u\dfrac{du}{dx} + e^x u = 0$

2. Find the explicit form general solution to the following first order ODEs.

(a) $\dfrac{du}{dx} + \dfrac{1}{x}u = x^2$ (b) $e^x\dfrac{du}{dx} - e^u = 0$

3. Find the explicit form general solution to these as well.

(a) $(u+x)\dfrac{du}{dx} + u + 3x = 0$ (b) $xu\dfrac{du}{dx} + x^2 - u^2 = 0$

Hint for (a): The quadratic formula may prove useful to determine the explicit form.

4. A tank is initially filled with 5 gallons of fresh water. A salt water solution flows into the tank at the rate of one gallon per minute. However, the in-flowing solution does **not** have constant concentration. It has variable concentration given by e^{-t} pounds of salt per gallon of water. There is another pipe which will output one gallon per minute of the fully mixed water/salt solution. Let $s(t)$ denote the amount of salt in pounds in the tank at time t minutes.

(a) Set up the first order initial value problem solved by $s(t)$.

(b) Solve for $s(t)$.

5. Find the general solution of the following second order differential equations by using the given factorization.

(a) $\left(\dfrac{d}{dx} + I\right)\left(\dfrac{du}{dx}\right) = \dfrac{d^2u}{dx^2} + \dfrac{du}{dx} = 0.$

(b) $\left(\dfrac{d}{dx} + I\right)\left(\dfrac{du}{dx} + u\right) = \dfrac{d^2u}{dx^2} + 2\dfrac{du}{dx} + u = x.$