Lecture 32. Gauss, the Prince of Mathematics

Mathematics is called the queen of sciences and Gauss was called the Prince of Mathematics.

Gauss and his life Carl Friedrich Gauss (1777-1855) is a German mathematician who contributed significantly to many fields, including number theory, statistics, analysis, differential geometry, geodesy, electrostatics, astronomy and optics. Sometimes known as the “prince of mathematics.”

Gauss was born in Brunswick, Germany. He was born into a poor and unlettered family. His father managed to earn a meager living through hard work as stonemcutter, gardener, canal worker and finally foreman for a masonry firm.
Carl was an extremely bright child; in later years he would joke, laughingly, that he could do computations in his head before he could talk. According to a well-authenticated story, \(^1\) one Saturday when his father was preparing the weekly payroll for his laborers, the 3 years old young Carl was looking over his shoulders. When his father wrote down the total amount of the payroll at the bottom, Carl chirped up, “Father, the reckoning is wrong; it should be...” True, the total that his father had written down was wrong and the young Gauss was correct. In later life he (Carl Gauss) loved to joke that he knew how to reckon before he could talk.

At the beginning of the year, to keep 100 pupils occupied, the elementary teacher, Büttner, and his assistant, Martin Bartels, assigned them the task of summing the integers from 1 to 100. Before the teacher barely finishied explaining the assignment, the 7 years old Gauss wrote the single number 5050 on his slate and deposited it on the teacher’s desk. Gauss has noticed that sum was 50 pairs of numbers each pair summing to 101. Astonished and impressed by the young student, Büttner arranged for Gauss to have special textbook, to have tutored by his assistant Martin Bartels.

In 1788 Gauss began his education at the Gymnasium \(^2\) with the help of Büttner and Bartels, where he learnt High German and Latin.

![Figure 32.2 Göttingen](image)

Gauss’ intellectual abilities attracted the attention of the Duke of Brunswick, who sent him first to the Collegium Carolinum (1792-95) in Brunswick and then to the University of Göttingen (1795-98). At the academy Gauss independently discovered Bode’s law, the


\(^2\)Gymnasium is a type of school providing secondary education in some parts of Europe.
He went to University of Göttingen as a student for three years, According to Bell \(^3\), “The three years (October, 1795  September, 1798) at the University of Göttingen were the most prolific in Gauss's life. The ideas which had overwhelmed Gauss since his seventeenth year were now caught - partly - and reduced to order. Since 1795 he had been meditating a great work on the theory of numbers. This now took definite shape, and by 1798 the Disquisitome Arithmeticaes (Arithmetical Researches) was practically completed.” At Göttingen, one of Gauss's teachers was Abraham Kaestner whom he used to ridicule quite often. Gauss called him “a poet among mathematicians and a mathematician among poets.”

![Figure 32.3](image)

At age 19, Gauss left Göttingen without a diploma, but by this time he had made one of his most important discoveries - the construction of a regular heptadecagon \(^4\) by ruler and compasses. This was the most major advance in this field since the time of Greek mathematics. Later, Gauss proved that a regular polygon of \(p\) sides where \(p\) is an odd prime, is constructible by straightedge and compass if \(p\) is of the form \(2^{2^k} + 1\). By the way, the values of \(2^{2^k} + 1\) for \(k = 0, 1, 2, 3, 4\) are

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3, 5, 17, 257, \text{ and } 65537
\]

which are all prime. Euler proved that when \(k = 5\), the value admits the factor 641. \(^5\)

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\(^3\)E.T. Bell, *Men of Mathematics.*

\(^4\)A regular polygon of 17 sides.

\(^5\)It is conjectured that \(2^{2^k} + 1\) is composite for all \(k \geq 5\) but there has yet no proof of this.
In 1799 Gauss returned to Brunswick to receive a degree. The Duke of Brunswick had agreed to continue Gauss’s stipend, and he requested that Gauss submit a doctoral dissertation to the University of Helmstedt. He already knew Pfaff, who was chosen to be his advisor. Gauss’s dissertation was a discussion of the fundamental theorem of algebra.

With his stipend to support him, Gauss did not need to find a job so that he devoted himself to research. He published the book *Disquisitiones Arithmeticae* in the summer of 1801, which is devoted to the number theory.

**Discovery of the dwarf planet** On January 1, 1801, astronomer Giuseppe Piazzi discovered Ceres, the smallest identified dwarf planet in the Solar System and the only one in the asteroid belt. Unfortunately, Piazzi had only been able to observe 9 degrees of its orbit before it disappeared behind the Sun. Ceres’ apparent position had changed (mostly due to the Earth’s orbital motion) so that other astronomers could not confirm the observations of Piazzi until the end of the year. However after such a long time it was difficult to predict its exact position. To recover Ceres, Carl Friedrich Gauss, then 24 years old, developed an efficient method of orbit determination. In only a few weeks, he predicted its path, and sent his results to von Zach. Zach published several predictions of its position, including one by Gauss which differed greatly from the others. When Ceres was rediscovered by Zach on 7 December 1801, it was almost exactly where Gauss had predicted. This success brought Gauss worldwide fame and let to the offer of an appointment at the St. Petersburg Academy, a post he declined.

![Figure 32.4 Ceres, Moon and Earth comparison](image)

Gauss was a prodigious mental calculator. When someone asked how he had been able to predict the trajectory of Ceres with such accuracy he replied, “I used logarithms.” The one then wanted to know how he had been able to look up so many numbers from the tables so
quickly. “Look them up?” Gauss responded. “Who needs to look them up? I just calculate them in my head!”

Living in Göttingen  Gauss married Johanna Ostoff in 1805. Despite having a happy personal life for the first time, his benefactor, the Duke of Brunswick, was killed fighting for the Prussian army. In 1807 Gauss left Brunswick to take up the position of director of the Göttingen observatory.

Gauss arrived in Göttingen in late 1807. In 1808 his father died, and a year later Gauss’ wife Johanna died after giving birth to their second son, who was to die soon after her. Gauss was married for a second time the next year to Minna the best friend of Johanna.

In 1809 Gauss published his second book, *Theoria motus corporum coelestium in sectionibus conicis Solem ambientium*, a major two volume treatise on the motion of celestial bodies. In the first volume he discussed differential equations, conic sections and elliptic orbits, while in the second volume, the main part of the work, he showed how to estimate and then to refine the estimation of a planet’s orbit. Gauss’s contributions to theoretical astronomy stopped after 1817, although he went on making observations until the age of 70. Gauss rarely left the city except on scientific business (In fact he never in his life leave the Germany, not even for a visit). From there, he worked for 47 years until his death at almost 78.
Here are some of Gauss’ contributions to mathematics and science.

- **Number theory**  At age 24, Gauss published one of the most brilliant achievements in mathematics, *Disquisitiones Arithmeticae* (1801). In it, Gauss systematized the study of number theory (properties of the integers). Gauss proved that every number is the sum of at most three triangular numbers and developed the algebra of congruences.

  In 1801, he proved the fundamental theorem of arithmetic, which states that every natural number can be represented as the product of primes in only one way.

- **Statistics**  The normal distribution, also called the Gaussian distribution, is an important family of continuous probability distributions, applicable in many fields. Gauss became associated with this set of distributions when he analysed astronomical data using them.

- **Analysis**  Gauss’ law, is a key result in the analysis of vector fields.

- **Differential geometry**  In differential geometry, a field of mathematics dealing with curves and surfaces, Gauss proved in 1828 an important theorem: the Theorema Egregium (remarkable theorem in Latin), establishing an important property of the notion of curvature. Informally, the theorem says that the curvature of a surface can be determined entirely by measuring angles and distances on the surface.

- **Non-Euclidean geometry**  Gauss arrived at important results on the parallel postulate, but failed to publish them. Credit for the discovery of non-Euclidean geometry therefore went to Janos Bolyai and Lobachevsky (see next lecture). However, he did publish his seminal work on differential geometry in *Disquisitiones circa superficies curvas*. He refused to publish any of it because of his fear of controversy.
• **Complex analysis**  He also discovered the Cauchy integral theorem for analytic functions but he did not publish it.

Gauss proved the fundamental theorem of algebra, which states that every polynomial has a root of the form $a+bi$. In fact, he gave four different proofs, the first of which appeared in his dissertation. Ironically, by today’s standard, Gauss’s own attempt is not acceptable, owing to implicit use of the Jordan curve theorem. However, he subsequently produced three other proofs, the last one in 1849 being generally rigorous. His attempts clarified the concept of complex numbers considerably along the way. Gauss solved the general problem of making a conformal map of one surface onto another.

![Figure 32.7 Gauss and his second wife.](image)

• **Geodesy**  In 1818 Gauss, putting his calculation skills to practical use, carried out a geodesic survey of the state of Hanover, linking up with previous Danish surveys. To aid in the survey, Gauss invented the heliotrope, an instrument that uses a mirror to reflect sunlight over great distances, to measure positions.

• **Electrostatics**  In conjunction with Wilhelm Weber, Gauss conducted research into magnetism and electricity and in 1833 invented an electromagnetic telegraph. He also devised a number of units for magnetic phenomenon and a unit for magnetic flux density is also named after him.

• **Astronomy**  In 1801, Gauss developed the method of least squares fitting but did not publish it. The method enabled him to calculate the orbit of the asteroid Ceres, which had been discovered by Piazzi from only three observations. Using his superior methods, Gauss redid in an hour’s time the calculations on which Euler had spent three days, and which sometimes are said to have led to Euler’s loss of sight in one eye. Gauss remarked unkindly: “I should also have gone blind if I had calculated in that fashion for three days.”
Unfortunate for mathematics? Bell thought it was unfortunate for mathematics that Gauss diverted his attention from it to other things. Had he continued exclusively in mathematics, he would have made many more revolutionary contributions in it.

Gauss remained active until his death. In his later years he focused more and more on applied mathematics. The appearance of his diaries and of some of his letters has shown that he kept some of his most penetrating thoughts to himself. He had discovered elliptic functions in 1800 and was in possession of the non-Euclidean geometry around 1816. Gauss seemed to be unwilling to venture publicly into any controversial subject.

He left many mathematical results unpublished but put them hidden in his diary. He wouldn’t publish anything which was not proven beyond any doubt and to his absolute satisfaction. He was absolutely meticulous about the beauty and completeness of a scientific theory. By comparing to Euler and Cauchy, Gauss ultimately published little, his collected mathematical works occupying only (!) 12 volumes. However his mathematical works are of such profundity that they have influenced the progress of the subject to the present.

Figure 32.8 Gauss