Earlier life  Évariste Galois (1811 - 1832) was a French mathematician born in Bourg-la-Reine, where his father was mayor. His mother was an educated woman and taught Galois at home until he entered school at the age of 12. Galois seems to have a happy childhood. There is no record of mathematical talent on either side of the family.

In 1823, at the age of 12, Galois was sent to school for the first time, entering the lycée of the Louis-le-Grand in Paris.

At first Galois did well in school and won prizes, but by his second year he became bored with the classical studies. His work became mediocre, and he had trouble with the school authorities. He clearly focused more on mathematics than other subjects, so it is no wonder that his school reports noted unsatisfactory progress in other subjects. Comments about his character being “singular,” “closed,” “not wicked,” “original and queer,” “argumentative,” “there is only slovenliness and eccentricity in his assigned tasks – when he deigns to pay
any attention to them,” and “He is wasting his time here, and all he does is torment his teachers and get into trouble.” In the school, Galois’ geometry textbook was the one by the mathematician Legendre. It was a difficult book, but he quickly mastered it. The algebra textbook used in the school disgusted him and he ignored it. It lacked, he said, the creator’s touch of a mathematician. Galois read Lagrange’s works on the theory of equations and analytic functions and Abel’s work.

**Attempted to apply for École Polytechnique**  When he was 16, Galois believed that he was ready to enter the École Polytechnique, the best university in France, but due to weak preparation on other subjects, failed the entrance exam.

Then Galois found a mathematics teacher, Louis Richard, and really started studying and doing mathematics. His first paper, on continued fractions, was published when he was 17. Gifted with the ability to carry out the most difficult mathematical investigations almost entirely in his head, Galois did not need help from teachers. Their insistence on details always left him exasperated, and he frequently lost his temper.

![Figure 39.2 École Polytechnique where Napoleon was Visiting in 1815.](image)

Galois was a mathematical genius. His most important mathematical works was formed when he was 17 years old, as we shall talk about soon. It is understandable that he was expected to go to the first rate college.

At 18, Galois reapplied to the École Polytechnique. It was normal practice that pupils could sit the École Polytechnique examinations at most twice so that Galois had to pass the exams this time. During the oral part of the exam, pupils were quizzed by two professors of the institution. Galois had a habit to calculate mostly in his head and to commit only the final results to the blackboard, which put him at a serious disadvantage. According to
one version\(^1\), when asked to outline the theory of arithmetical logarithms, Galois informed the examiner arrogantly that there were no arithmetical logarithms. Legend has it that in his frustration with the examiners' inability to understand his non-standard methods, he threw the blackboard eraser at one examiner's face. As a consequence, this was the end of his attempt to enter the École Polytechnique. In historian E.T. Bell's words, compared to Galois, the two examiners were “not worthy to sharpen his pencils.”\(^2\) Two decades later, Terquem\(^3\) remarked, “A candidate of superior intelligence is lost with an examiner of inferior intelligence.”

![Figure 39.3 Revolution, 1830, France](image)

**In the college** At 19 years old, Galois gained admission to the École Normale. But he wasn’t a happy student. His revolutionary ideals collided with those of the Normale, so he joined the revolutionary parties of the school and in 1830 was expelled. His father died soon after.

With no stipend and no father, Galois had no money to live on. Later, Galois put up a notice in the front of a grocery shop opposite the university, announcing a private class in higher algebra meeting once a week with him as the tutor. Some students came, but after a while they found the subject way over their heads, and they stopped coming.


\(^3\)Olry Terquem (1782-1862) was a French mathematician, best known for his work in geometry. He was among the first who recognized the importance of the work of Évariste Galois.
With no job, no school, no money, Galois devoted all his energies to revolutionary politics, and writing mathematical memoirs on higher algebra.

He joined the National Guard —— “If a carcass is needed to stir up the people, I will donate mine.” Galois was jailed for supposedly threatening the King, but was found ‘not guilty’ by a jury. Finally he was convicted and sentenced to 6 months in jail for “illegally wearing a uniform.”

**Theory of algebraic equations**  What Abel had proved is that in general, an equation of degree higher than 4 has no solution by radicals. On the other hand, many special equations were solvable by radicals. The characterization of these remained an open problem. Galois continued Abel’s work to study this problem, and he definitely answered what specific equations of a given degree admit an algebraic solution.

Galois’ work was of great importance. It not only solved the above problem, but also introduced the first time the concept of “group” which has lots of applications today.

However, Galois’ result was available in print 14 years after his death. The reason is a combination of bad luck and negligence described as follows.

In May 1829, when he was only 17 years old, Galois submitted his results on the algebraic solution of equations to the Academy of Sciences. Augustin Cauchy was appointed as a referee. Cauchy either forgot or lost the communication.

In February 1830, Galois submitted a new version of his paper to for a competition in Grand Prize in mathematics. Joseph Fourier took the manuscript to his home to read, but died before writing a report about them and the papers were never found.
In January 1831, Galois submitted his paper again to the Academy of Sciences. After six months, it was rejected by the referee Simeon Poisson who remarked: 4

His arguments are not sufficiently clear, nor developed enough for us to judge their correctness.... It is hoped that the author would publish his work in its entirety so that we can form a definite opinion.

Galois was understandably upset: “Genius is condemned by a malicious social organization to an eternal denial of justice in favor of fawning mediocrity.”

When Galois was finally released from the jail, his last misadventure began. “Thus it happened that he experienced his one and only love affair. In this, as in everything else, he was unfortunate. Galois took it violently and was disgusted with love, with himself, and with his girl.” A few days later Galois encountered some of his political enemies and “an affair of honor,” a duel, was arranged. In those days, the practice of settling disputes by duels was very common, and not only in France. 5

![Figure 39.5 Duel](image)

**Figure 39.5** Duel

**In the eve of the duel** On the eve of the duel, Galois knew he had little chance to win, so he spent all night writing the mathematics which he didn’t want to die with him, often writing “I have not time. I have not time” in the margins. Among the proofs he wrote

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5The theory has been advanced that the challenger was hired by the police, who arranged the confrontation to eliminate what they considered to be a dangerous radical. See D. Burton, The History of Mathematics, sixth edi, McGraw-Hill Higher Education, 2007, p.335.
in those last busy hours was the solution to a riddle that had tortured mathematicians for centuries:

“Under what conditions can an equation be solved using radicals? ”

He wrote to a friend a summary of his discoveries in the theory of equations. This pathetic document, in which he asked his friend to submit his discoveries to the leading mathematicians, ended with the words:

“You will publicly ask Jacobi or Gauss to give their opinion not on the truth, but on the importance of the theorems. After this there will be, some people who will find it to their advantage to decipher all this mess.”

Figure 39.6 Galois’ handschrift.

He also wrote on that same night:

“I did several new things concerning analysis. Some of them are about the theory of equations, others about integral functions. Concerning the theory

of equations, I have tried to find out under what circumstances equations are solvable by radicals, which gave me the opportunity of investigating thoroughly, and describing, all transformations possible on an equation, even if it is the case that is not solvable by radicals.

He sent these results, as well as the ones the Academy had lost, to his friend Auguste Chevalier, and, on May 30, 1832, went out to duel with pistols at 25 paces.

Galois was shot in the intestines, and was taken to the hospital. He comforted his brother with “Don’t cry, I need all my courage to die at twenty.” He died the day after the duel and was buried in an unmarked, common grave, at the tragically young age of 21.

Chevalier and Alfred Galois (Evariste’s younger brother) later sent the papers to Gauss and Jacobi, but there was no response.  

Figure 39.7 Galois’ tomb.

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Galois’ work published  Twenty four years after Galois’ death, Joseph Liouville edited some of Galois’ manuscripts and published them with a glowing commentary.

“I experienced an intense pleasure at the moment when, having filled in some slight gaps, I saw the complete correctness of the method by which Galois proves, in particular, this beautiful theorem: In order that an irreducible equation of prime degree be solvable by radicals it is necessary and sufficient that all its roots be rational functions of any two of them.”

Galois’ complete works fill only 60 pages, but he will be remembered. Struik wrote⁸: We may speculate on the possibility that if Galois had lived, modern mathematics might have received its deepest inspiration from Paris and the school of Lagarange rather than from Göttingen and the school of Gauss.

Figure 39.8  Part of the letter written on the eve of the duel.