MATH 2331 EXAM 2 REVIEW NAME______________

November 2, 2017

NO CALCULATORS!

1. Find the eigenvalues and eigenvectors of the following matrices. Determine whether each matrix is diagonalizable. For those that are diagonalizable, find P such that $P^{-1}AP$ is diagonal. 32 pts

a. $A=\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

b. $A=\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

c. $A=\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

12 pts

2. Let $B=\left\{\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix},\begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}\right\}$, and $\boldsymbol{x}=\begin{bmatrix} -2 \\ 2 \\ -2 \end{bmatrix}$ 10 pts

Assume **x** is in the span of B. Find the coordinates of **x** with respect to the basis B.

3. If $\boldsymbol{A}=\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)=\mathbf{A}\begin{bmatrix} x \\ y \end{bmatrix}$, and $D=\{(x,y):\ 0\le x\le 1,\ 0\le y\le 1\}$:

a. Sketch T(D) and label the vertices. 4 pts

b. What is the area of T(D)? 6 pts

4. Let $A=\begin{pmatrix} 1 & 3 & 3 & 3 & 4 \\ 2 & 6 & 5 & 7 & 10 \\ -1 & -3 & -4 & -2 & -2 \end{pmatrix}$. Find a basis for each of Nul(A), Col(A), Row(A), and Nul(A^T). 24 pts

5. Find $\begin{bmatrix} x \\ y \end{bmatrix}$ to minimize $\left\|\mathbf{A}\begin{bmatrix} x \\ y \end{bmatrix}-\mathbf{b}\right\|^2$, where $\mathbf{A}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 0 \\ 0 & 2 \end{bmatrix}$ and $\mathbf{b}=\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$. 20 pts

10 pts

6. a. Is it possible for a nonhomogeneous system of four equations in six unknowns to have a unique solution for some right-hand side? Explain. 6 pts

b. Is it possible for such a system to have a solution for every right-hand side? Explain. 6 pts

7. Consider this symmetric stochastic matrix:

$$A = \begin{bmatrix} .8 & .2 \\ .2 & .8 \end{bmatrix}$$

a. What are the eigenvalues and eigenvectors of A? 12 pts

b. Find the coordinates of $u_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ with respect to the basis of eigenvectors found in part a. 10 pts

c. Find $\lim_{k\to\infty} A^k u_0$ 12 pts

8. A 3x3 symmetric matrix has a null space of dimension one containing the vector (1,1,1). Find bases and dimensions of the column space, row space, and left null space.

9. Determine whether $\{v, u, w\}$ is linearly dependent or independent.

$v = (2,\ -2,\ 3)$ $u = (3\ \ 0,\ 4)$ $w = (1,\ -4,\ 2)$ 12 pts

10. Define "basis". Use a complete sentence (or two). 10 pts

11. Define "linearly independent". Use a complete sentence (or two) 10 pts

12. Find the projection matrix **P** that projects any vector $\mathbf{v} \in \mathbf{R}^3$ onto the line generated by $\mathbf{b} = (2,\ -1,\ 2)^T$. Find the eigenvalues and eigenvectors of **P**. 15 pts

Textbook supplementary Exercises:

Ch 3, p. 285: 1ℓ-p.
Ch 4, p. 262: 1, 3, 5, 7, 9, 11, 12, 13, 14, 15.
Ch 5, p. 326: 1, 3, 5, 7, 9, 12, 13, 14, 17.
Ch 6, p. 390: 1a-q, 3, 4, 5, 6, 7,