MATH 2331 FINAL EXAM NAME_______________

December 12, 2011 ID #_______________

NO CALCULATORS!

1. a. Find the complete solution to the system:

$$\mathbf{BX} = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 2 & 2 \\ 2 & 2 & 2 & 4 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$$

12 pts

b. Find a basis for the column space of B. Then express each column of B as a combination of the basis. (Hint: B has rank 2) 12 pts

c. Find an equation $a_1b_1 + a_2b_2 + a_3b_3 = 0$ for all vectors $\mathbf{b} = (b_1, b_2, b_3)$ for which **BX** = **b** has a solution. 12 pts

3. Let $T: \mathbf{R}^3 \to \mathbf{R}^3$ be a linear transformation such that: $T(1,0,0) = (1,2,3)$, $T(0,1,0) = (4,5,6)$, $T(0,0,1) = (7,8,9)$. Find the matrix for T. *Hint: Compute T(x,y,z).* 9 pts

4. a. Find the eigenvalues and eigenvectors of each of these matrices. Identify which are invertible and/or diagonalizable.

$A = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$ 6 pts

$B = \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$ 6 pts

$C = \begin{bmatrix} 4 & 2 \\ 4 & 2 \end{bmatrix}$ 6 pts

$D = \begin{bmatrix} 5 & 5 \\ 1 & 1 \end{bmatrix}$ 6 pts

$E = \begin{bmatrix} 2 & 1 \\ -17 & 4 \end{bmatrix}$ 6 pts

$F = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$ 6 pts

b. Organize the matrices A-F in disjoint sets of similar matrices. 8 pts

5. a. Find the matrix P_1 that projects (x,y,z) onto the line through (1,2,2). 10 pts

b. Find three eigenvalues and three independent eigenvectors for P_1. 12 pts

c. Find the matrix P_2 that projects (x,y,z) onto the plane orthogonal to (1,2,2). 15 pts

6. This problem uses least squares to find the curve $y = ax + bx^2$ that best fits these 4 points in the plane: $(x_1, y_1) = (-2,2)$, $(x_2, y_2) = (-1,1)$, $(x_3, y_3) = (1,0)$, $(x_4, y_4) = (2,2)$.

a. Write down 4 equations $ax_i + bx_i^2 = y_i$, i = 1, 2, 3, 4, that would be true if the line actually went through all four points. 8 pts

b. Now write those four equations in the form $\mathbf{A}\begin{bmatrix} a \\ b \end{bmatrix} = \mathbf{y}$ 4 pts

c. Now find $\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix}$ that minimizes $\left\| \mathbf{A}\begin{bmatrix} a \\ b \end{bmatrix} - \mathbf{y} \right\|^2$. 14 pts

7. Consider this symmetric Markov matrix:

$$A = \begin{bmatrix} .4 & .6 \\ .6 & .4 \end{bmatrix}$$

a. What are the eigenvalues of A? 8 pts

b. Find $\lim_{k \to \infty} \mathbf{A}^k \mathbf{u_0}$ with $u_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ 12 pts

8. $$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1+x & 1+x & 1+x \\ 1 & 1+x & y+x & y+x \\ 1 & 1+x & y+x & y+x-z \end{bmatrix}$$

Find $\det(\mathbf{A})$. 12 pts