MATH 2331 FINAL EXAM NAME________________
December 14, 2009 ID #________________

NO CALCULATORS!

1. Find the complete solution to the system:

$$B = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & -1 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

12 pts

2. If you know that $\det(\mathbf{A}) = 2$, where $A = \begin{bmatrix} \text{row 1} \\ \text{row 2} \\ \text{row 3} \end{bmatrix}$, and

$$\mathbf{B} = \begin{bmatrix} 3(\text{row 1}) - (\text{row 2}) + 4(\text{row 3}) \\ 5(\text{row 2}) - 11(\text{row 3}) \\ 7(\text{row 3}) \end{bmatrix}$$, what is det(**B**)?

12 pts

3. Determine whether the following set of vectors is linearly dependent or independent.

$v = (2, \ -2, \ 3)$ $u = (3 \ \ 0, \ 4)$ $w = (1, \ -4, \ 2)$ 12 pts

4. Define "basis". Use a complete sentence (or two). 10 pts

5. Define "linearly independent". Use a complete sentence (or two) 10 pts

6. Organize these matrices into disjoint sets, where all of the matrices in each set are similar.

$A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$ $D = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ $E = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$ $F = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$G = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$

18 pts

7. Find the characteristic equation of P, if

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

16 pts

8. A 3x3 symmetric matrix has a null space of dimension one containing the vector (1,1,1). Find bases and dimensions of the column space, row space, and left null space.

15 pts

9. For what vectors $\mathbf{b}$ does the system $\mathbf{Ax}=\mathbf{b}$ have a solution, if $\mathbf{A}=\begin{bmatrix}1 & 2 & 5\\ 0 & -1 & -1\\ 5 & 3 & 18\end{bmatrix}$? Find an equation for $\mathbf{b}$: $c_1b_1+c_2b_2+\ldots+c_nb_n=0$

12 pts

10. A 3 by 3 matrix A has eigenvalues and eigenvectors: $\mathbf{Ax}_1=\mathbf{x}_1,\ \mathbf{Ax}_2=2\mathbf{x}_2,\ \mathbf{Ax}_3=3\mathbf{x}_3$. Suppose $\mathbf{b}=4\mathbf{x}_1+5\mathbf{x}_2+6\mathbf{x}_3$

a. Find $\mathbf{y}=c_1\mathbf{x}_1+c_2\mathbf{x}_2+c_3\mathbf{x}_3$ so that $\mathbf{A}^2\mathbf{y}-3\mathbf{Ay}+\mathbf{4y}=\mathbf{b}$ (Find c_1, c_2, c_3). 12 pts

11. Find $\begin{bmatrix}x\\ y\end{bmatrix}$ to minimize $\left\|\mathbf{A}\begin{bmatrix}x\\ y\end{bmatrix}-\mathbf{b}\right\|^2$, where $\mathbf{A}=\begin{bmatrix}1 & 0\\ 0 & 1\\ 2 & 0\\ 0 & 2\end{bmatrix}$ and $\mathbf{b}=\begin{bmatrix}1\\ 2\\ 3\\ 4\end{bmatrix}$. 20 pts

12. Find the eigenvalues and eigenvectors of A.
Is A diagonalizable? Invertible?

a. $A=\begin{pmatrix}-1 & 5\\ 5 & -25\end{pmatrix}$. 12 pts

b. $A=\begin{pmatrix}-4 & 5\\ -5 & 4\end{pmatrix}$. 12 pts

c. $A=\begin{pmatrix}2 & 3\\ -3 & 8\end{pmatrix}$. 12 pts

13. Find the projection matrix **P** that projects any vector $\mathbf{v}\in\mathbf{R}^3$ onto the line generated by $\mathbf{b}=(2,\ -1,\ 2)^T$. Find the eigenvalues and eigenvectors of **P**. 15 pts