Solution to 1 using laplace

$x'' + 2x' + 10x = 2\cos(2t) \qquad x(0) = 0 \qquad x'(0) = 0$

$$L(x'' + 2x' + 10x) = L(2\cos(2t))$$
$$L(x'') + 2L(x') + 10L(x) = 2L(\cos(2t))$$
$$s^2L(x) + 2sL(x) + 10L(x) = 2L(\cos(2t))$$
$$L(x)(s^2 + 2s + 10) = \frac{2s}{s^2+4}$$
$$L(x) = \frac{2s}{(s^2+4)(s^2+2s+10)} = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+2s+10}$$

$$\rightarrow \quad 2s = (As+B)(s^2+2s+10) + (Cs+D)(s^2+4)$$
$$\rightarrow 0s^3 + 0s^2 + 2s + 0 = (A+C)s^3 + (2A+B+D)s^2 + (10A+2B+4C) + (10B+4D)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 \\ 10 & 2 & 4 & 0 & 2 \\ 0 & 10 & 0 & 4 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/13 \\ 0 & 1 & 0 & 0 & 4/13 \\ 0 & 0 & 1 & 0 & -3/13 \\ 0 & 0 & 0 & 1 & -10/13 \end{array}\right] \begin{array}{l} = A \\ = B \\ = C \\ = D \end{array}$$

$$L(x) = (A)\frac{s}{s^2+4} + \left(\frac{B}{2}\right)\frac{2}{s^2+4} + (C)\frac{(s+1)}{(s+1)^2+3^2} - \left(\frac{C}{3}\right)\frac{3}{(s+1)^2+3^2} + \left(\frac{D}{3}\right)\frac{3}{(s+1)^2+3^2}$$

$$x = (A)\cos(2t) + \left(\frac{B}{2}\right)\sin(2t) + (C)e^{-t}\cos(3t) + \left(\frac{D-C}{3}\right)e^{-t}\sin(3t)$$

$$\boxed{x = \frac{3}{13}\cos(2t) + \frac{2}{13}\sin(2t) - \frac{3}{13}e^{-t}\cos(3t) + \frac{-7}{13}e^{-t}\sin(3t).}$$