Math 3331 Homework Solutions

Assignment due 9/4.

1. For the initial value problem

$$y'' + 2y' + 10y = 0, \quad y(0) = 2, \quad y'(0) = 3$$

find an equivalent initial value problem for a first-order system

Solution: Let $u_1 = y$, $u_2 = y'$, then

$$u_1' = u_2$$

$$u_2' = y'' = -10y - 2y' = -10u_1 - 2u_2$$

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So the equivalent initial value problem is

$$u_1' = u_2 \qquad u_1(0) = y(0) = 2$$

$$u_2' = -10u_1 - 2u_2 \qquad u_2(0) = y'(0) = 3$$

Use a numerical solver (pplane) to plot the solution to this initial value problem in $-5 \le y \le 5$, $-5 \le y' \le 5$

(Use pplane with 'Keyboard input' for initial value:

— See attached.

#1

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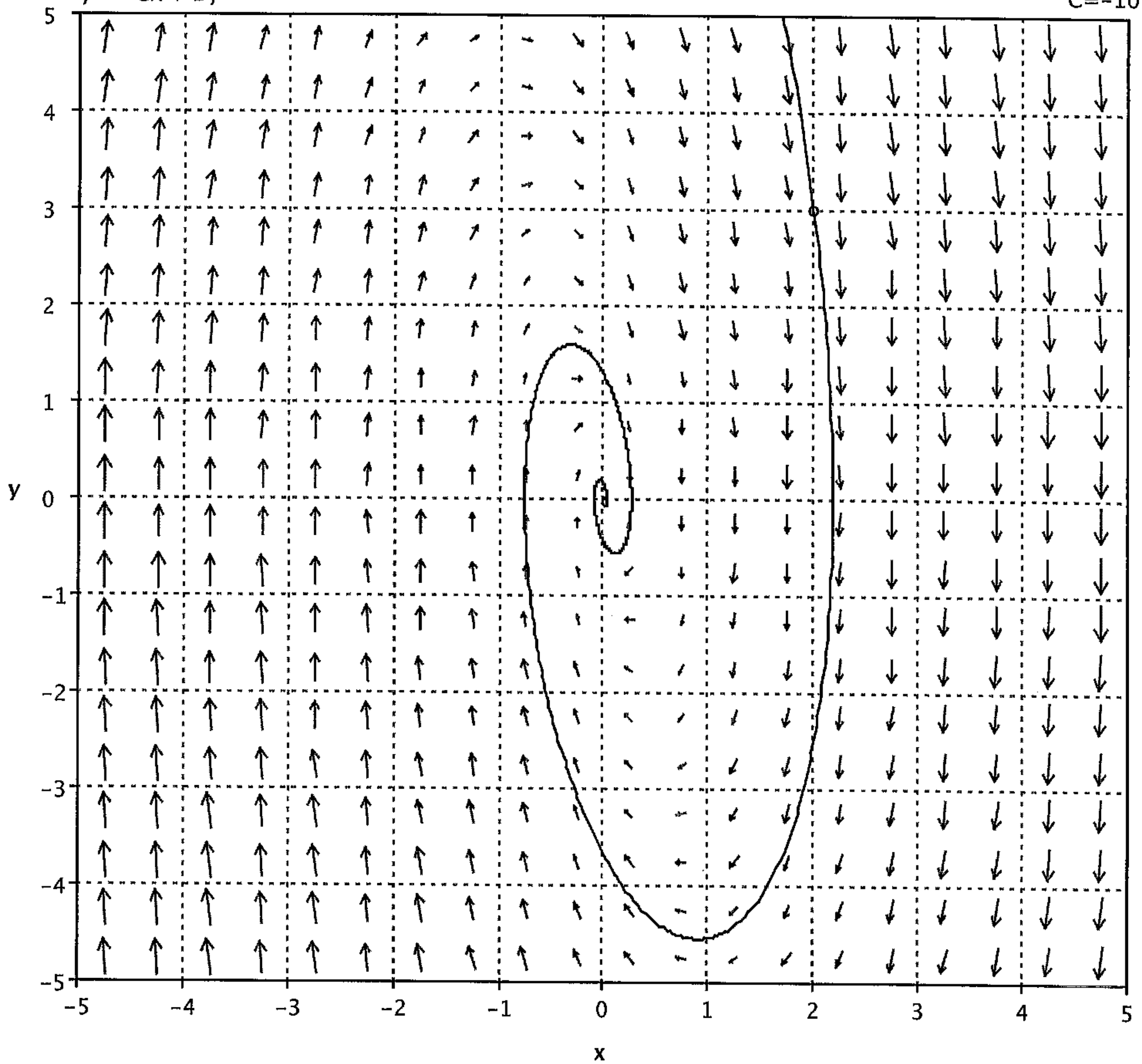
$x' = Ax + By$

$y' = Cx + Dy$

A=0 B=1

C=−10 D=−2



$X'' + 2X' + 10X = 0$ as $\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -10 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$

$x(0) = 2$

$y(0) = 3$

#2

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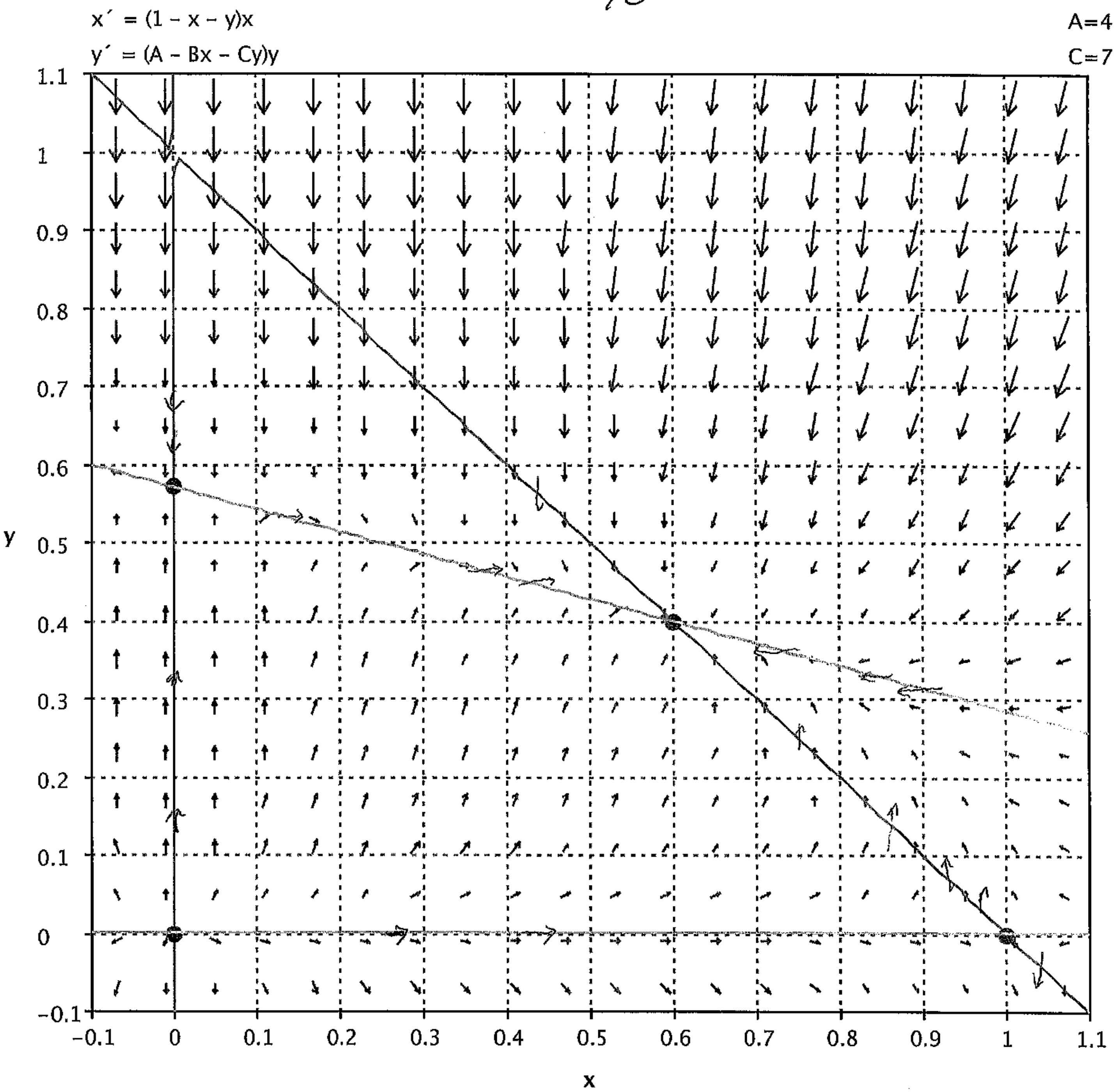

x′ = (1 − x − y)x
y′ = (A − Bx − Cy)y
A=4
C=7
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
y
−0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
x

#3

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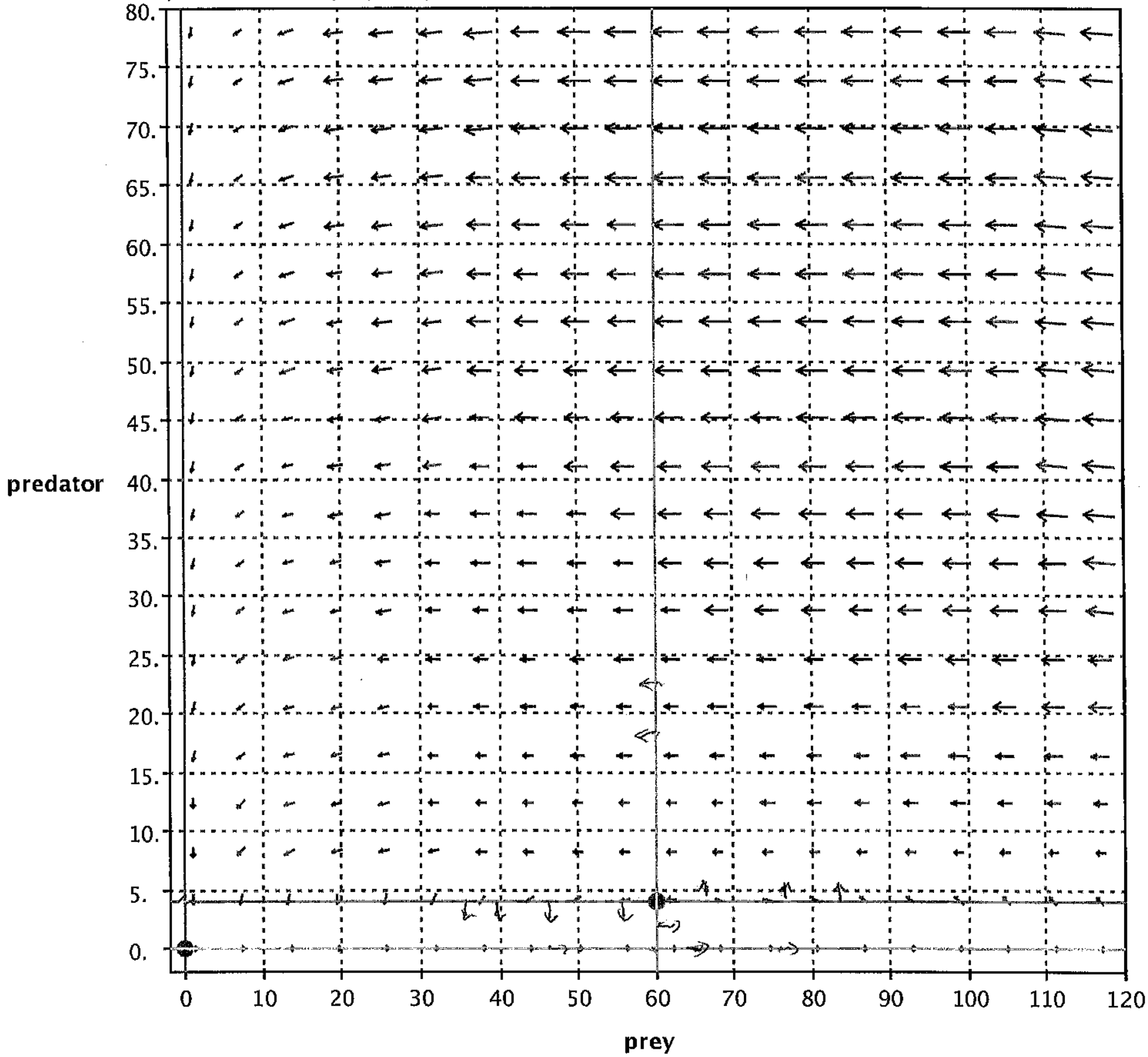
prey′ = (A − B*predator)prey

predator′ = (D*prey − C)predator

B = 0.1 A=.4 B

D = .005 C=.3 D



This is the correct solution as the problem was posed

#3

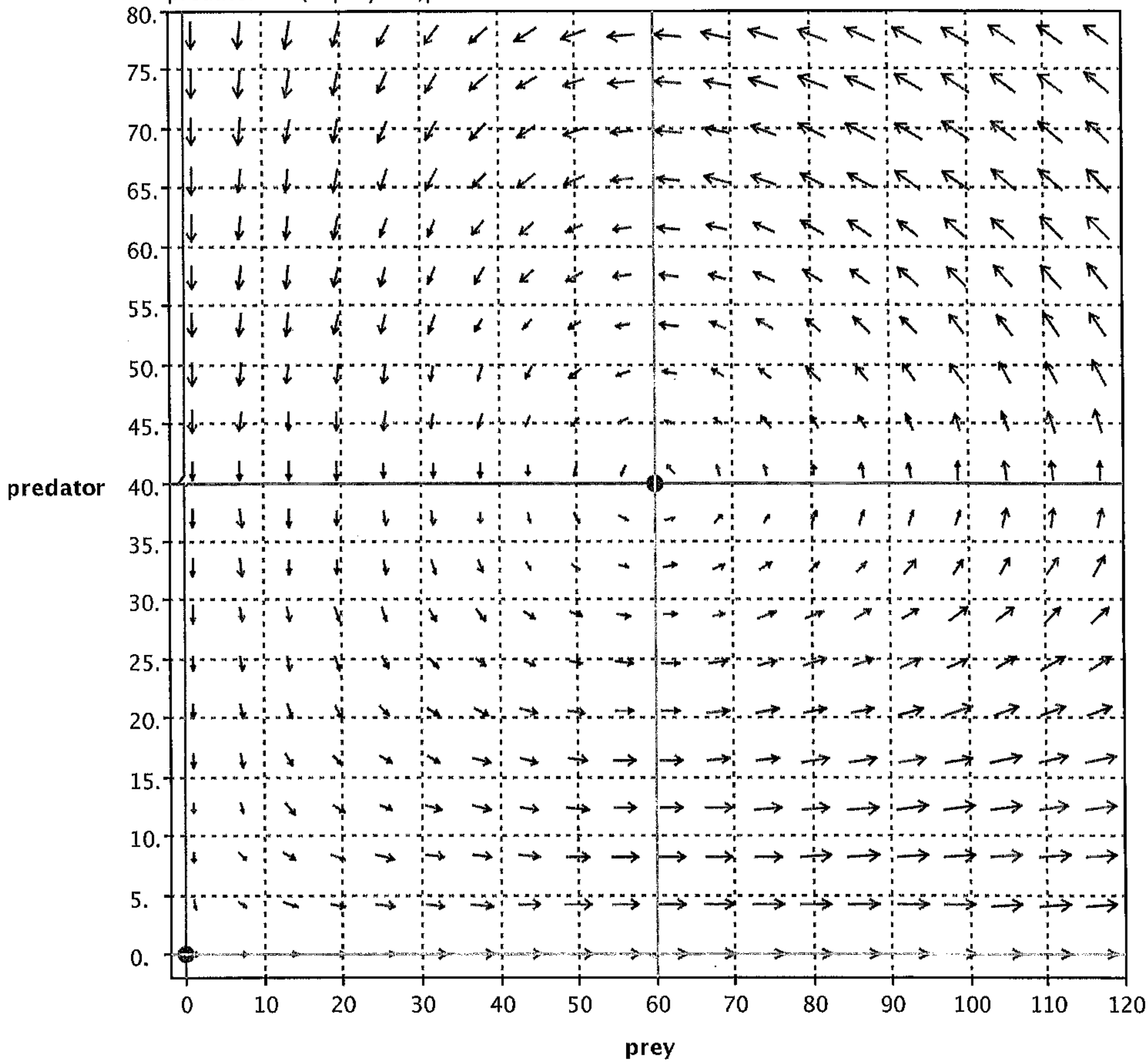
prey′ = (A − B*predator)prey

predator′ = (D*prey − C)predator

B=.01
B=.005

A=.4 B

C=.3 D



this is what I intended, B=.01

Math 3331 Homework Solutions

Assignment due 4/11

4. Put this system in the form: $\vec{x}'(t) = A(t)\,\vec{x}(t) + \vec{f}(t)$.
Identify any values of t_0 or $\vec{x}_0$ for which an initial condition $x(t_0) = x_0$ does not satisfy the requirements of the Existence and Uniqueness Theorem (Theorem 3.2 on page 348).

$$x_1'(t) = 3t\,x_1 - \sin(t)\,x_2 + t^2 x_3 + \cos(2t)$$
$$x_2'(t) = x_1 + 5t^2 x_2 + \tfrac{1}{t} x_3 + e^t$$
$$t\,x_3'(t) = t\,x_1 - x_3 + 2.$$

Solution

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 3t & -\sin t & t^2 \\ 1 & 5t^2 & \frac{1}{t} \\ 1 & 0 & -\frac{1}{t} \end{pmatrix}}_{A(t)} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \underbrace{\begin{pmatrix} \cos 2t \\ e^t \\ 2/t \end{pmatrix}}_{\vec{f}(t)}$$

$A(t)$ and $f(t)$ are not continuous at $t=0$.
Any initial condition $X(t_0) = X_0$ satisfies the requirements of Theorem 3.2 except at $t_0 = 0$.